

SP

PERIODICA POLYTECHNICA

ELECTRICAL ENGINEERING — ELEKTROTECHNIK

MAGYAR TUDOMÁNYOS AKADÉMIA
KÖTELEKES FIZIKAI ÉS MATEMATIKAI INTÉZETE
SPEKTRÁLIS FIZIKA OSZTÁLYÁNAK

VOL. 1 * NO. 2 * 1957 KÖNYVTÁRA



POLYTECHNICAL UNIVERSITY
TECHNISCHE UNIVERSITÄT
BUDAPEST

PERIODICA POLYTECHNICA

Contributions to international technical sciences published by the Polytechnical University, Budapest (Hungary)

Originalbeiträge zur internationalen technischen Wissenschaft, veröffentlicht von der Technischen Universität, Budapest (Ungarn)

PERIODICA POLYTECHNICA

includes the following series

Engineering
Electrical Engineering
including Applied Physics
Chemical Engineering

enthält folgende Serien

Maschinen- und Bauwesen
Elektrotechnik und angewandte Physik
Chemisches Ingenieurwesen

The issues of each series appear at quarterly intervals

Einzelnummern der genannten Serien erscheinen vierteljährlich

CHAIRMAN OF THE EDITORIAL BOARD — HAUPTSCHRIFTFLEITER

Z. CSÜRÖS

EDITORIAL BOARD — SCHRIFTFLEITUNG

**O. BENEDIKT, S. BORBÉLY, L. GILLEMOT, P. GOMBÁS, L. HELLER,
K. P. KOVÁCS, J. PROSZT, G. SCHAY, L. VEREBÉLY, I. VÖRÖS**

EXECUTIVE EDITOR — SCHRIFTFLEITER

J. KLÁR

The rate of subscription to a series is \$ 4,00 a year. For subscription or exchange copies please write to

Jahresabonnement pro Serie: \$ 4,00. Bestellungen und Anträge für Tauschverbindungen sind zu richten an:

PERIODICA POLYTECHNICA

BUDAPEST 62, POSTAFIÓK 440

WIDE-ANGLE IMAGE FORMING SYSTEMS

(DISTORTION-FREE PANORAMIC PROJECTION)

N. BÁRÁNY

Institute for Instrumental Design and Precision Mechanics of the Polytechnical University,
Budapest

(Received March 28, 1957)

Introduction

The problem of a large field of vision or a wide image angle has always played an important part in the development of optical instruments. Wide-angle objectives are being used in a certain special class of photography only, hence, less attention is paid to their improvement as is the case with telescopes where a large field of vision is still a critical factor, particularly for opera glasses, field glasses, etc. However, considerable difficulties have been encountered in connection with the lens corrections required for large relative apertures.

It has been attempted, in the course of time, to realize in practice all the suggestions put forward by those skilled in the art, and to apply in the design of both telescopes and photographic objectives all available technical improvements. The author's aim was to give a brief chronological summary of only the most important types of instruments.

Wide-angle projection has recently come into the foreground once more. This has induced the author to set forth some ideas with respect to wide-angle and panoramic projection in connection with the phenomenon of after-images in human vision.

I. General

Optical systems forming real images may be grouped into two categories, when considered from the viewpoint of the path of rays. In the case of image formed by means of photographic lenses the path of rays is discontinuous, whereas it is continuous for telescopes and microscopes.

Images formed by photolenses may be received on various kinds of screens, such as ground glass plates, or photosensitive layers, etc. Although the image is viewed on the ground glass plate, it is the photographic procedure that will fix the image. The photographic picture representing the scene as viewed in the moment of exposure is, of course, permanent and available at any time. Incident rays are more or less dispersed by the grainy surface of the ground glass so that the image is produced by means of dispersed radiation. Actually, one may call the procedure one of double image formation, as the crystalline

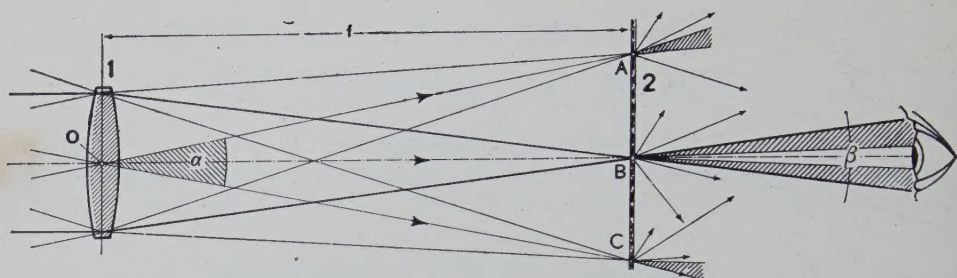


Fig. 1. Intermittent path of ray, illustrating the principle underlying photographic cameras. The image becomes visible to the eye, the radiation being dispersed by the opaque screen. The path of rays and the field of image are indefinite.

lens reproduces a second retinal image from the one originally formed by the photolens and received on the screen (Fig. 1). Only a portion of the dispersed radiation issuing from the image points A, B and C formed by the chief rays indicated by arrows reaches eye 3. Thus, in order to cover the entire image formed on a large-size ground glass plate not only the eye but also the head must be turned. Only the central part of the image can be seen distinctly. When observing the image on the glass plate, the *objective* image is produced by objective lens 1, and the *subjective* image by the crystalline lens. The ground glass plate plays no part in the latter procedure, hence the path of rays and, in consequence, image formation, is not continuous but intermittent. Both these images are real ones. The angle α enclosed by the straight lines connecting the centre O (that is, the posterior principal point) of lens 1 with points A and B at the margin of the picture but still included in the perception range of the screen, is called *image angle* of the lens for photographic cameras, and *field of vision* for telescopes and microscopes.

If the image received on the ground glass plate of the camera is viewed through magnifying glass 3 placed behind it (Fig. 2) while removing the glass plate from the path of rays, one finds that the picture, instead of disappearing,

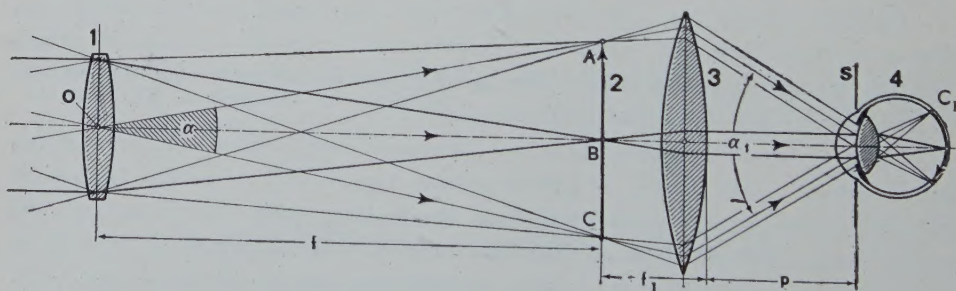


Fig. 2. Continuous path of rays. The principle underlying telescopes, microscopes and most optical instruments. The image is produced by the objective in the aperture of the screen limiting the field of vision. The path of rays and the field of vision behind the image are definite, that is, directed

remains visible. Thus the camera has been converted into a telescope, with photolens 1 acting as objective lens, and magnifying glass 3 acting as eye-piece. The pencils of rays are directed by the lenses, in strict conformity with the laws of optics, to eye 4 on whose retina the subjective image A_1, C_1 is formed. In this type of system the image is produced by *directed* beams, and not by dispersed ones. The telescopic image can, of course, be seen only as long as one looks into the telescope. In general, the image ABC cannot be reproduced; nevertheless, it is possible, though not usual, to photograph the said image.

The angle of vision α of the image formed by objective lens 1 (Fig. 2) is called *real* field of vision, expressed in terms of angular measure. This image, subject to the magnifying power of the telescope, is seen through magnifying glass 3 at an angle α_1 , representing the *virtual* field of vision of the telescope. Objective lens 1, eye-piece 3 and crystalline lens of eye 4 make a complete optical system. The design of such an instrument must, therefore, take into account the optical properties of the eye as well as psychological factors. The path of rays suffers no interruption in its passage from the objective lens to the retinal image, hence, image formation is *continuous*. Consequently, the individual elements for image formation, such as lenses, spherical mirrors, etc. cannot be positioned at will. The same applies to the visual distance p (representing the distance between the eye and the eye-piece), it being imperial for obtaining a full panoramic view of the complete field of vision, as well as for exploiting the luminous intensity of the optical system that the eye be placed to the intersection of the beams emerging from the system at the smallest section point, that is to plane S of the exit pupil. Optically, this means that the exit pupil of the system must coincide with the entry pupil of the eye. As a result of the aberrations in image formation of the lenses, the exit pupil is, in general, not a plane surface but a surface of higher order, with a maximum luminous intensity in the centre, gradually fading off toward the margins. This phenomenon may well be observed in connection with common opera glasses.

Obviously, telescopic observation is a direct process dependent upon time, whereas photographic representation is independent of time. Photography produces indirectly visible pictures, since the so-called *latent* image generated on the negative by radiation can only be made visible by inserting a separate step, that is, by development into a negative picture, and then by a subsequent positive procedure. The negative picture permits making as many copies as necessary. Any period of time may lapse between exposure, development and copying. A telescopic image, on the other hand, can only be seen while looking into the telescope. Perception and generation of the image take place simultaneously while the instrument is placed in front of the eye. In other words, a telescope only performs its function when placed before the eye. The same applies to magnifying glasses, microscopes, and other optical instruments subject to similar principles of optics.

Let us remove glass plate 3 from the path of ray (Fig. 1) while observing the image. Image ABC will continue to be seen, unless accommodation of the eye to the image distance in question is changed. The ability of maintaining accommodation may easily be acquired by some practice. In this case, eye 3 accommodated to point B will perceive only the pencils of rays of an angular field β whereas further pencils, indicated in the figure by dotted lines, intersecting one another at the other points and travelling towards the image plane, do not even reach the pupil of the eye.

Examination of Figs. 1 and 2 will facilitate distinction between continuous and discontinuous paths of rays. In the case of Fig. 1 only a small portion of the "directed" rays can reach the eye. Hence, it is impossible for the stationary eye to command the view of the entire image, whereas in the case illustrated in Fig. 2 all the pencils received by eye-piece 3 travel towards the eye. Thus the continuous path of rays secures simultaneous observation of the entire picture.

In accordance with the wave propagation of radiating energy, the process of representation (image formation) is similar for all optical systems. The differences are restricted to the path of rays, and are noticeable in connection with observation only.

The objective and subjective efficiency of any optical systems is determined by

1. magnification
2. field of vision
3. luminous intensity
4. resolving power
5. contrasts
6. psychological factors.

All these are closely interrelated in accordance with the laws of geometrical optics as well as those of the wave theory of radiation.

In the following observations we will confine ourselves to the investigation of the *visual field* in order to ascertain what may be expected from the latest developments in optical systems and their application in practice in the various spheres of use. This category of apparatus comprises the various types of projectors suited for displaying to a number of observers the picture obtained by means of either photographic cameras, microscopes, or telescopes. In this field the so-called cinematoscopic process meant an important step forward, whereas the problem of large-surface or wide-angle projection including the extreme case of panoramic (180°) projection field by optical and mechanical means still remains to be solved. The author will attempt to find the answer by taking into account the psychological reactions of the eye, too.

In our investigations, we shall first be concerned with photographic cameras, then with telescopes, and, finally, with apparatus comprising both kinds of systems, from optomechanical as well as historical point of view.

II. Wide-angle photographic cameras

Leonardo da Vinci invented the *pinhole camera* or camera obscura, a simple device by which, in principle, images of up to 180° angles may be produced without the use of optical elements such as lenses and spherical mirrors. Here,

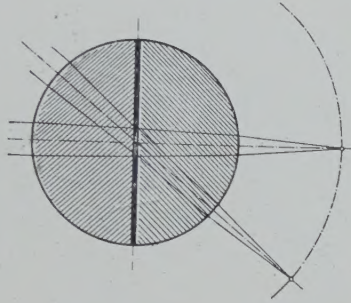


Fig. 3. A simple lens comprising two cemented hemispheres, with a screen of permanent aperture in the centre

the basis of image formation is the phenomenon of interference of radiating energy, the image point being the *Airy disc* possessing a certain diameter, thus being measurable. This phenomenon may be explained by the *Huyghens* element-

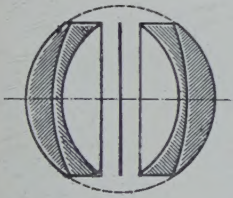


Fig. 4. The *Harrison* lens. A wide-angle photographic lens of permanent relative aperture (1860)

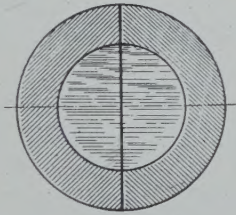


Fig. 5. The *Sutton* lens. A wide-angle spherical shell lens of permanent relative aperture, filled with water

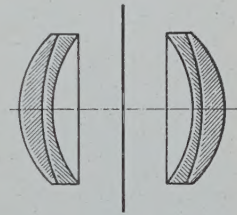


Fig. 6. The *Steinheil* lens. A wide-angle aplanat (1860)

ary spherical waves. The pinhole camera is the only optical device working by the principle of wave theory, as it produces a mosaic-like image consisting of Airy discs. The diameter of the aperture of the pinhole camera is dependent on the image distance, that is the distance between aperture and image. For a pinhole camera adjusted to infinity, image distance may be termed focal length, a designation used in connection with lenses. The smaller the diameter of the aperture, the shorter the image distance and vice-versa. However, owing to refraction apt to impair good definition, the diameter of the aperture must not be reduced beyond a certain limit. The smaller the image distance the larger the object space to be covered, in other words, the image angle. Theoretically,

it would be possible to increase the said angle up to 180° , if it were not for technical difficulties.

In view of the wave theory image distance for a given diameter of aperture and wave length is

$$k = \frac{R^2}{\lambda}$$

where K represents the image distance, R the radius of the aperture and λ the wave length of radiating energy in $\text{m}\mu$. According to this formula, every wave length has a corresponding image distance, so that a picture taken by means of colourless radiation of various wave lengths cannot be sufficiently acute.

The major conclusion to be drawn from the above formula is that it is possible to compute image distances for apertures of any diameter size. Let the diameter be 60 mm ($R = 30$) and the wave length of greenish-yellow radiation, best perceived by the eye, be $\lambda = 10^{-6} \cdot 555 \text{ m}$, the necessary image distance will be

$$k = \frac{R^2}{\lambda} = \frac{30^2}{10^{-6} \cdot 555} = 1,621 \text{ km.}$$

Thus, the image distance obtained even for such a small aperture is very inconvenient, not to mention the extremely poor luminous intensity. This drawback may be obviated by introducing some optical elements, such as a lens, a spherical mirror, or a mirror glass in order to shorten the image distance, thereby increasing luminous intensity. Thus it is not the lens but the aperture that matters for image-forming devices, the lens being used to reduce inconvenient image distances or to increase luminous intensity to the required extent, respectively. Aware of these two factors, researchers have long been concerned with the idea of producing photographic lenses of large image angles coupled with a convenient luminous intensity.

The first embodiment of this idea was a 19th century spherical lens of the aplanatic type, consisting of two hemispheres and separated by a diaphragm. If the parallel pencils of rays incident at different angles are so directed that their optical axes intersect the centre of the diaphragm, the pencils of rays will travel through the lens without refraction. The image thus obtained is on a spherical surface, of a radius f . (Fig. 3)

The 1860 HARRISON-lens (Fig. 4) consists of two opposite, symmetrical achromatic doublet lenses of steep curvature. The outer surface of the two symmetrical members may be covered by a spherical surface, with the diaphragm in between. This lens arrangement has a relative aperture of $1:36$ and gives fairly acute photos of a 90° angle. Owing to spherical aberration, caused by strong curvature, achromatization of the system is rather difficult so that the stop number must be quite large.

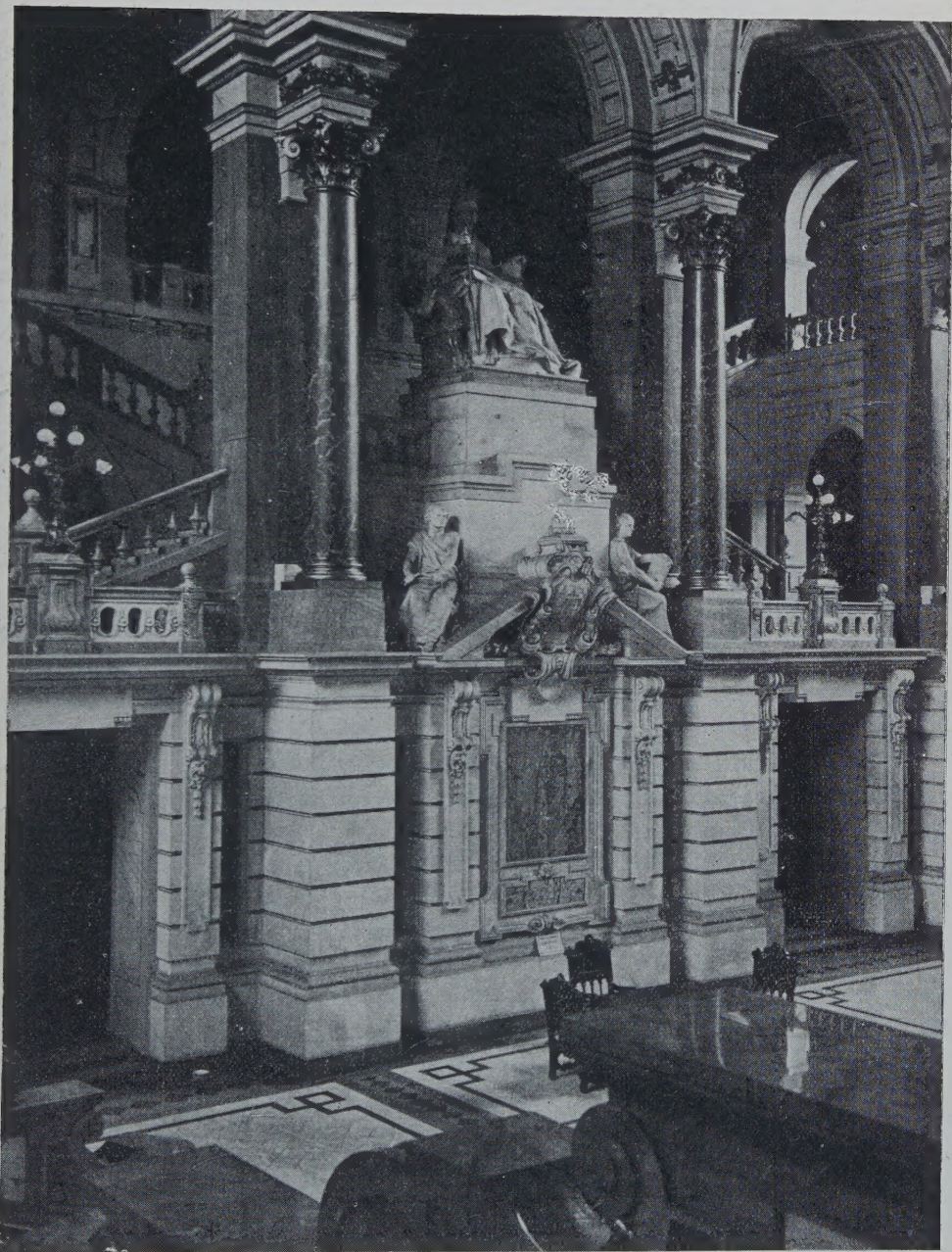


Fig. 7. Photography taken of the interior of a large hall. Exposure : 4 minutes (at noon in June).
Relative aperture 1 : 22

Let us mention here *Sutton's* spherical lens (Fig. 5), a system of a certain historical interest, composed of two cemented spherical shells and filled with water.

The *STEINHEIL* wide-angle aplanatic lens (Fig. 6) dates back to 1866. It is a four-component lens of two meniscus-shaped symmetrical doublets separated by a diaphragm. The Steinheil lens eliminates coma, distortion and chromatic differences in magnification. It has an almost 100° field angle for a relative aperture of $1 : 7$ and $f/9$ cm. The picture shown in Fig. 7 was taken with a Steinheil lens.

Wide-angle lenses produce pictures of extremely marked perspective, that is, close objects appear to be very large in proportion to those in the background,

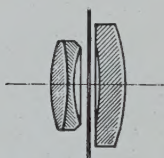


Fig. 8. The *Busch* Pentagonal, a wide-angle lens (1903)

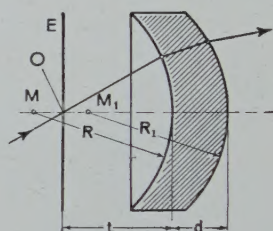


Fig. 9. The principle of an aplanat meniscus by *E. v. Hoegh* (1900)

and straight lines are highly convergent. Such lenses are well suited for taking pictures of monumental effect, in addition to interiors.

The *BUSCH* "Pentagonal" lens seen in Fig. 8 was designed in 1903. It has a 120° angle for a maximum relative aperture of $1 : 18$, if the slight loss of acuity at the margins is neglected.

The 1900 *HOEGH aplanatic meniscus* is of scientific interest. This lens formed the basis for the *GOERTZ "Hypergon"*, a lens of the widest angle so far known. This system was born under circumstances worthy of note. In the 1900's there was a heated scientific argument going on between Dr. *PAUL RUDOLPH* of the *Jena Zeiss Works* and *EMIL VON HOEGH* of *Goertz, Friedenau*, in connection with the elimination of image curvature and astigmatism. Contrary to *RUDOLPH's* views, *HOEGH* based his assumption on the *PETZVAL-principle*.¹

The method of correction of curvature for photographic systems is set by the *PETZVAL* formula, the practical application of which was proved by *von HOEGH* using a simple lens of zero curvature (Fig. 9), bounded by two equal surfaces of radius R . If the lens has a thickness d and a refractive index n , then

$$f = \frac{n}{d} \frac{R^2}{[n - 1]^2}.$$

¹ Archiv f. wissensch. Photographie II., 1900.

Owing to the magnitude of R^2 , f is always positive, and inversely proportional to thickness d . In practice, this type of lens may always be constructed with a rather short focal length, without having to increase considerably its thickness. Taking a lens of $R = 12$ mm, $f = 100$ and $n = 1.6$, then

$$d = \frac{nR^2}{f[n-1]^2} = 6.4$$

Von Hoegh has proved that this type of lens, if used in combination with a front-stop, produces anastigmatic and plane images, similarly to landscape-lenses. Fig. 9 shows the two surfaces of a radius of curvature $R = R_1$, the centres of curvature being M and M_1 . The lens has a thickness d , and the stop E is at a distance t from the lens. The original data stated by von HOEGH are

$$R = 11,993$$

$$R_1 = 11,900$$

$$t = 7,1212$$

$$d = 6,0903$$

Thus, the radii of the meniscus lens are almost equal, and $f = 100$.

In accordance with the PETZVAL formula, this lens has a radius of infinite length, thus it produces a plane image, as the reciprocal value of the radius of curvature for plane images is for any lens represented by the formula

$$\frac{n-1}{n} \left[\frac{1}{R} - \frac{1}{R_1} \right]$$

The pencil of infinitesimal diameter passing through the axial centre O of the screen E is refracted to the right by the front surface. Subsequently, the pencil becomes divergent, so that the astigmatic image points produced by refraction are virtual. The back surface further refracts the pencil to the right, the angle of incidence being positioned opposite to the perpendicular of incidence. Following the last refraction, the emerging beams become convergent, forming almost coincident real astigmatic image points. Such beams may be called homocentric. Applying the above consideration, freedom from astigmatism as well as from curvature may be obtained, since the image point formed by the homocentric beam falls just into the focal plane. Taking an angle $\alpha = 30^\circ$, astigmatic image surfaces will not diverge from the focal plane by more than 0.25 mm, and the distance of astigmatic image points, also called astigmatic difference, will fall off to zero. Sagittal and tangential surfaces retreat from the image plane in opposed directions.

VON HOEGH maintains that for every anastigmatic system the image may be made plane by conducting oblique incident main pencils through the system with at least two refractions of similar direction, one of the two refracting surfaces causing the pencil to diverge, and the other to converge.

For the application of the HOEGH formula, however, the "freedom" implied by the *Petzval* formula must be taken into account, that is, its application must have quite distinct limits, particularly as regards the determination of sagittal and tangential surfaces.

A two-meniscus combination, based on the HOEGH computations, was designed and produced by Messrs. GOERTZ, under the designation *Hypergon*



Fig. 10. The Hypergon of E. v. Hoegh. A wide-angle photographic lens comprising two aplanat menisci, based on the *Goertz* principle

(Fig. 10). This lens suffered a certain set-back in the course of time but, owing to its excellent properties, its production was lately resumed. The Hypergon consists of two similar menisci, arranged symmetrically in relation to the stop, and covers a 140° field.



Fig. 11. The Hypergon, with the Stoke air-driven starwheel placed before the lens arrangement

The *Voigtländer Collinear* is a wide-angle photolens, a symmetrical lens arrangement with a relative aperture of $1:12.5$.

It should be noted that in order to cover the entire space of a 12 by 18 cm picture a photolens of 21 cm focal length is generally required. The focal length of commercial camera lenses equals the diagonal of the rectangular negatives.

The beams emerging at wide angles from the lens arrangements described above cause illumination to fall off considerably toward the margin of the picture. Thus, the ratio of illumination between centre and margins is $1:8$ for the Hyper-

gon. Various methods of correction are known. A front-lens similar to a filter glass, gradually deepening to darkness from centre to the margins (enixsantos-glass) is provided in the *Rodenstock 120° field Pentagonal*.

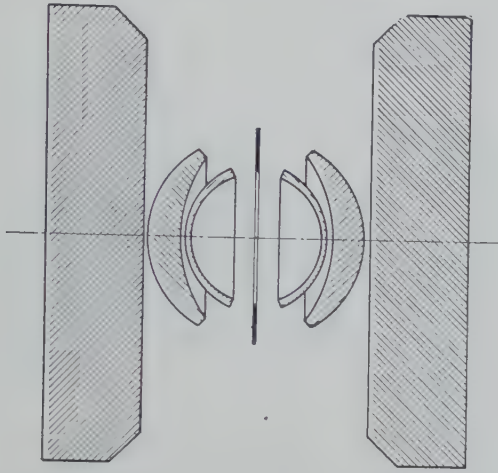


Fig. 12. The Topogon of Zeiss, a wide-angle lens for topographic surveying

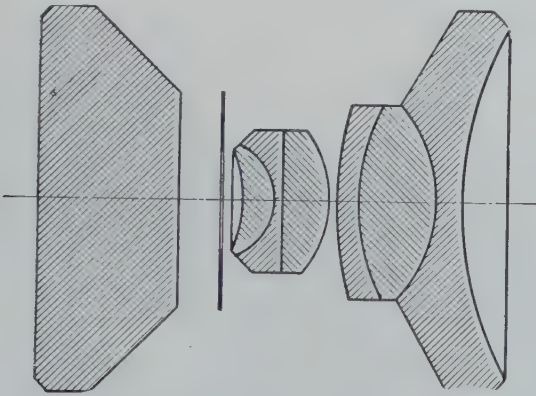


Fig. 13. The British Topogon lens

The Hypergon comprises a *Stocke* rotary screen (star wheel) inserted in front of the lens for good illumination (Fig. 11). This screen is rotated by a current of air, generated formerly by a rubber ball, whereas in up-to-date lens arrangements the current of air is generated by an adjustable clock-work automatism. The Hypergon works in two steps. Exposure must be eight times the unscreened exposure time corresponding to the selected stop number, when illuminating the margins of the negative with the rotary screen. The screen is then removed from the lens by means of a spring-operated arm fixed on the mount, and the normal exposure time as required for the centre of the negative is applied. This two-step operation requires a certain skill but in modern cameras the adjustment

of the two exposure times as well as rotation and removal of the screen is effected automatically. The relative aperture is 1 : 3,1.

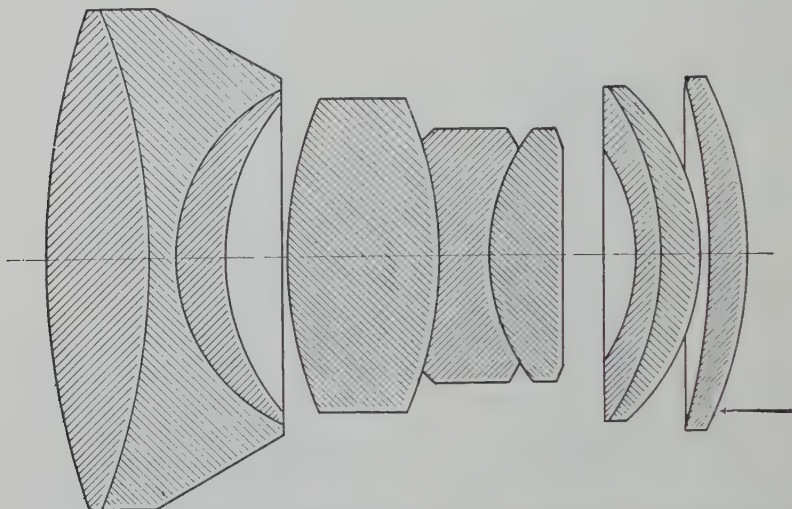


Fig. 14. L. Berthele's Aviatar for topographic surveying (System Wild)

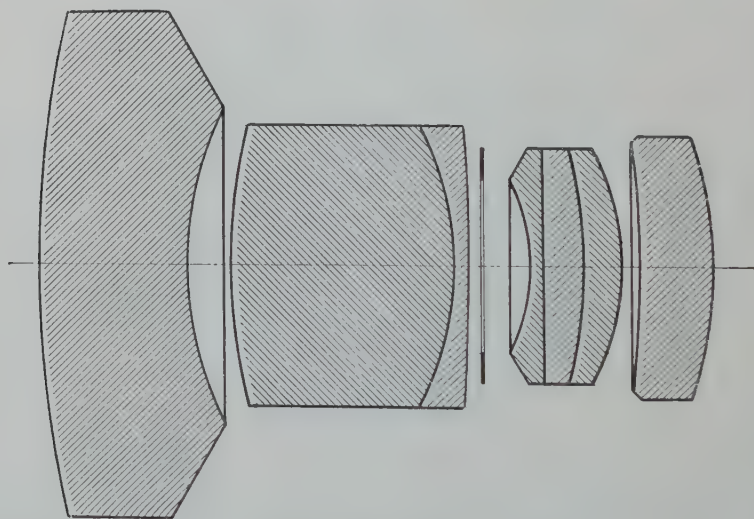


Fig. 15. Biogon of W. Merte. A wide-angle lens of large relative aperture. (System Zeiss)

The width of the angle, of course, gives rise to a certain amount of distortion at the margins which is often erroneously ascribed to the lens. In fact, this phenomenon is the result of the law of central projection, and has nothing to do with the optical aberrations of the lens arrangement. In any event, it is advisable to avoid taking photographs of arc-shaped, cylindrical or spherical objects, in situated close to the margin.

The relative aperture of wide-angle lenses is generally rather small, yet with proper illumination they allow of snapshots, too. It has been attempted to produce systems of this type with larger relative apertures.

The *Zeiss Topogon* (Fig. 12) used for topographical takings was constructed in 1935. It has a relative aperture of 1:6.3, and covers a field of 100°. The meniscus-like opposed lenses are bounded, in front and in the rear, by plane-parallel glass plates.

The British photolens of the *Topogon* type represented in Fig. 13 has a relative aperture of 1:6, covering again a field of 100°. The 150 *Aviotar* ($f/17$ and $f/21$ cm) for aerial reconnaissance (Fig. 14) was designed by *Berthele* for Messrs. *Wild*. It covers a field of 60°.

The *Zeiss Biogon* $f/35$ mm for commercial cameras shown in Fig. 15 belongs to the same category of lenses. It has a relative aperture of 1:2.8 and covers a 60° field.

It must be pointed out that, in connection with lens arrangements of large relative aperture but of short focal length, designers encounter serious difficulties when attempting to eliminate aberrations, not to mention heavy weight and high costs of production. It has been suggested to substitute them — if not for the purpose of image formation, but for the projection of light — by combinations of prisms or mirrors, such as the *Fresnel* ring lens arrangement² made of pressed glass. It was constructed as long ago as 1820. The central directing lens is surrounded by ring lenses of different curvatures and ring prisms of different refractive angles. Radiation is parallelized partly by refraction partly by total reflection. The *Fresnel* lens was originally built for lighthouse purposes. In recent ring lens systems the radii of curvature change from zone to zone. This system, too, made of pressed glass: the central luminous source is a small incandescent lamp. The system produces practically parallel, soft but intense radiation.

The zonal mirror designed by *WEBER* for Messrs. *I. D. MOLLER* secures even more accurate paths of rays. It consists of spherical mirrors of gradually increasing focal lengths fitted into one another. The system works as a mirror, the pencils being reflected by the surfaces. Combining the principles of the *Mangin* and *Schmidt* mirrors, it is used for purposes of projection. It has a 100 mm focal length and a 86 mm diameter, with a nearly 1:1 relative aperture.

Speaking of mirrors, mention must be made of the lowest-priced and simplest wide-angle photographic device, the common glass ball. The picture is extremely distorted yet the objects can be distinguished, and with some skill, it is possible to correct the distortion. In emergencies, it can be used for the topographical taking of interiors, etc. It has the advantage of freedom from chromatic aberration, the virtual image obtained by reflection.

² A. FRESNEL, *Projet d'un phare à feux tournants dans lequel réflecteurs seraient remplacés par des lentilles*. Oeuvres compl. III., 73—79. Paris. Imprim. imper. 1870.



Fig. 16. A wide-angle picture taken using the rear element of a Tessar lens. Part of the picture, indicated in the Figure, can be well-used if the picture has been properly screened

Photography is similarly possible by using spherical mirrors of short focal lengths and of large diameters. The image formed by parabolic mirrors is real and, after magnification, the central part of the picture can be well-used. The disadvantage of both spherical and concave mirrors is, in addition to distortion, that part of the camera will unavoidably appear in the picture, covering a certain portion of the objects to be photographed.

In the following a practical method is described for transforming the conventional triplet lens arrangements of cameras into wide-angle systems.

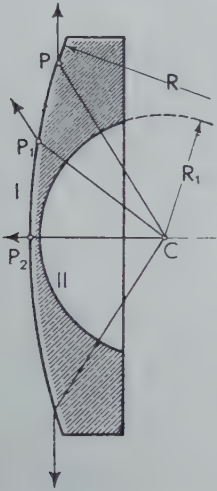


Fig. 17. The Hill panoramic front lens arrangement with inverted path of rays.

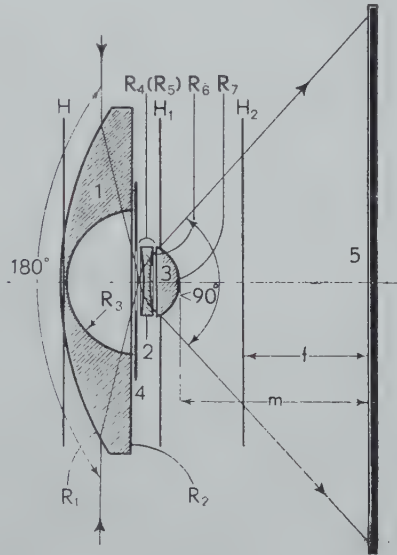


Fig. 18. The Hill panoramic lens arrangement. Field angle 180°

The method consists in dismounting the first member, usually comprising a positive and a negative element in one mount, and taking the picture with only the two-component cemented achromatic back lens. This can most conveniently be done with the *Tessar* and *Heliar* type lenses. Such an operation will, naturally, entirely upset the well-balanced correction of the system, thus the back component will be subjected to heavy curvature, chromatic aberration, astigmatism and coma. By adjusting to sharp definition either the centre or the margin, the unadjusted portion will be so blurred as to be hardly distinguished. It is therefore suggested to proceed in the following manner: the camera is adjusted to the position within the limits of dullness where the whole picture will be uniformly dull. This may be corrected by heavy screening, and a yellow filter will free the image from chromatic aberration.

The lens covers a field of 90°, thus the base board of the camera will appear in the picture (Fig. 16).

For a long time, no wide-angle lens was able to compete successfully with

the *Hypergon*. Only after 25 years, in 1924, did the ingenious invention of the Englishman ROBIN HILL succeed in increasing the angle of field to 180° and above.³

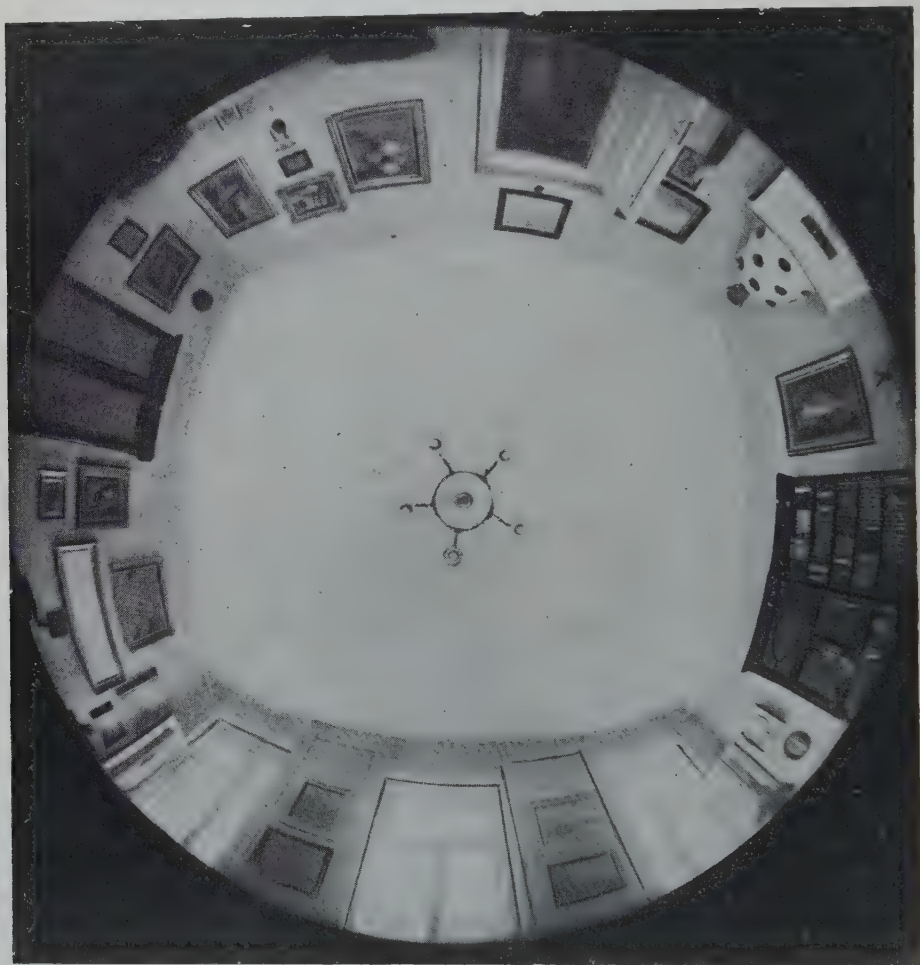


Fig. 19. Photography of an interior, taken with the lens arrangement represented in Fig. 18

For purposes of explanation, let us invert the path of rays (Fig. 17). The rays from the curvature centre C of the surface having a radius R_1 of the highly dispersive lens 1 pass the back surface without refraction. On emerging at points P, P_1 of the front surface having a radius L , the rays are refracted. It is clear

³ British patent granted on the 4th December, 1924. Robin Hill "Camera for photographing the whole sky". Quart. Journ. Meteor. Soc. 50 (1924), 227. — Manufactures by R. and Beck Ltd. 69 Mortimer Street, London W. 1.

that the axial ray suffers no refraction when emerging at P_2 . The extreme emerging rays include a 180° angle. This angle of view may be widened still further, up to 220° by appropriately varying the lens sizes. Thus, a so-called "rear-view" lens is produced. The negative lens will yield only a virtual image, hence, if one

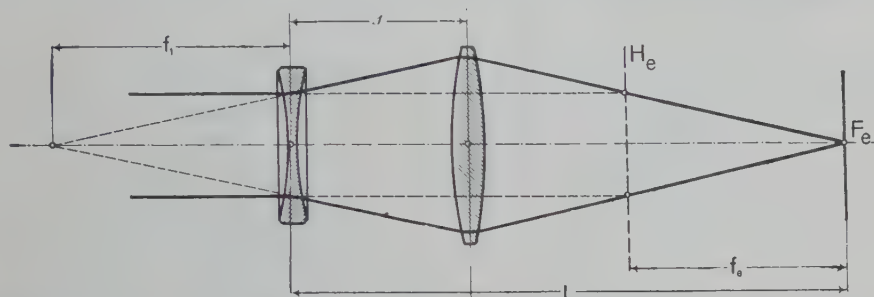


Fig. 20. Path of rays in the "inverted" teleobjective. Owing to the resultant principal plane situated at the rear, the mechanical length of the system is greater than the resultant focal length

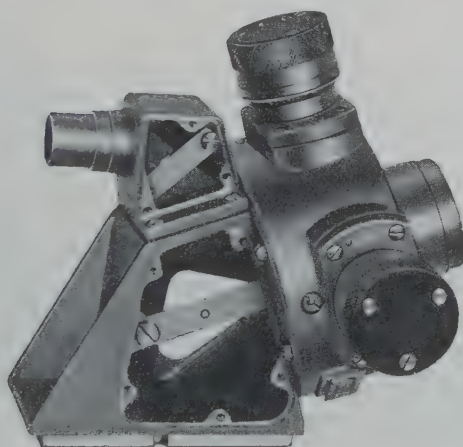


Fig. 21. The lens arrangement represented in Fig. 20, applied in a telescope of broken sight line. The resultant principal plane is inside the large-size lower prism

intends to produce real images, it must be combined with a positive system consisting of elements 2 and 3 (Fig. 18). The lens has a negative diameter of 58 mm, and a relative aperture of $1:22$ will produce a panoramic image covering a 6,5 by 9 cm plate (Fig. 19). The adjustable stop 4 is inserted right behind the dispersing lens 1.

The system has a focal length of 33–46 mm. Reverting to Fig. 18, H is the rear principal plane of the meniscus, H_1 the front principal plane of the positive components, H_2 the rear principal plane of the whole arrangement. It is from this latter plane that the focal length is computed. Thus, the focal length f for the whole system is shorter than the intercept length m , i. e., the

distance from the rear surface of the last lens 3 to the negative 5. The arrangement reminds one of an inverted teleobjective, where the resultant focal length is, however, longer than the intercept length, so that, owing to the considerable focal length, the extension of the camera is correspondingly short. In the case of teleobjectives, the negative lens is at a certain distance behind the positive component facing the objects space. Let the negative component be placed *in front of* the positive component; by intersecting the incident and emergent rays (Fig. 20) the resultant principal plane H_e and the resultant focal length

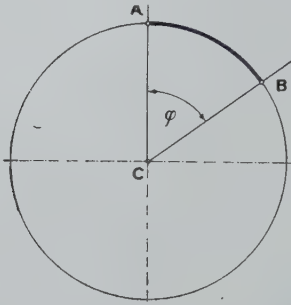


Fig. 22. A sphere or part of it, observed from the centre under an angle φ

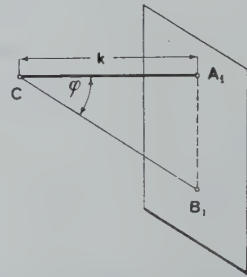


Fig. 23. The points represented in Fig. 22 projected on a plane surface, as observed under an angle φ

F_e are obtained. In that case, as in the *Hill* lens, the resultant focal length f will be shorter than the full length L of the system. The length of the path of rays due to the inverting prism arrangement would prevent the application of wide-angle positive lenses for e. g. telescopes. It has been found possible to increase the mechanical length of the system by drawing the negative lens closer. In prismatic telescopes (Fig. 21) constructed according to this principle both the negative and positive lenses are fixed in a short tube provided on the left hand side on top. Behind it there is a right-angled 45° prism, and under it a penta roof prism constituting the inverting system.

Reverting to the *Hill* lens, it must be stated that the photographs taken with this lens suffer heavy distortion, particularly at the margins. As a method of analyzing the said distortion, it is advisable to investigate the different ways of projecting a sphere or parts thereof on a plane surface (Fig. 22).

Let the observer be placed at the centre C of a circle of radius R . He will then view points A and B at an angle φ . Fig. 23 shows the projections A_1 and B_1 of the afore-said points A and B , seen by the observer from a distance k at an angle α . Perspective, in other words representation, would be true if the angular distance of central observation were equal to the angular distance of the projected points, that is,

$$\varphi = \alpha$$

or, accordingly

$$\operatorname{tg} \varphi = \operatorname{tg} \alpha.$$

It is, however, impossible to fulfil this condition for a plane image of finite size. One has to resort to other methods which, although introducing distortion

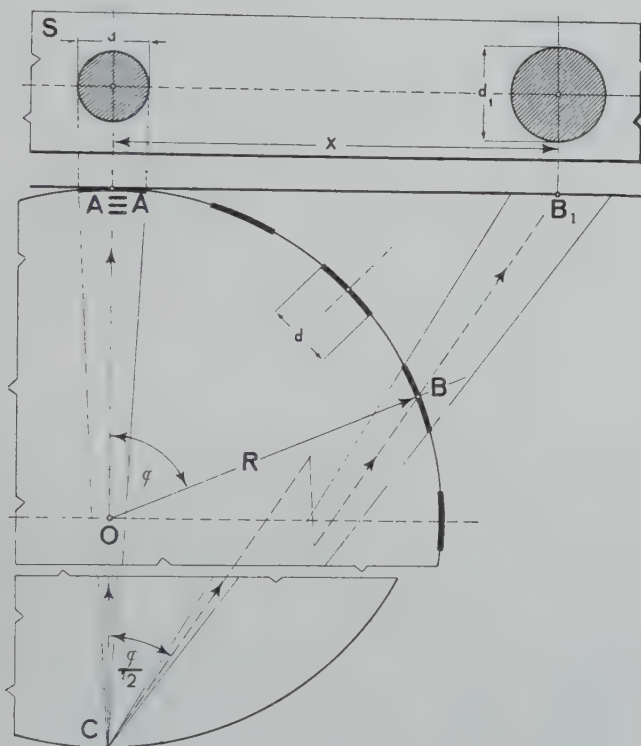


Fig. 24. The principle of stereographic projection

of the perspective, allow of the representation of a hemisphere in a finite plane. The following relations are characteristic of the various methods of projection.

1. For stereographic projection :

$$\operatorname{tg} \frac{\varphi}{2} = A \operatorname{tg} \alpha.$$

2. For equidistant projection :

$$\varphi = A \operatorname{tg} \alpha.$$

3. For orthographic projection :

$$\sin \varphi = A \operatorname{tg} \alpha.$$

$$\frac{d\varphi}{1 + R \cos \varphi} = d\omega$$

$$DE = 2R d\omega$$

$$\frac{2R}{AB} = \cos \frac{\varphi}{2}$$

$$AB = \frac{2R}{\cos \frac{\varphi}{2}}$$

$$DE = 2R \frac{d\varphi}{R[1 + \cos \varphi]} = \frac{2 d\varphi}{2 \cos^2 \frac{\varphi}{2}} \quad BF = \frac{2R}{\cos \frac{\varphi}{2}} \cdot \frac{d\varphi}{2}$$

$$\frac{BF}{BC} = \cos \frac{\varphi}{2}$$

$$BC = \frac{BF}{\cos \frac{\varphi}{2}} = \frac{2R d\varphi}{\cos^2 \frac{\varphi}{2}}$$

$$DE = \frac{d\varphi}{\cos^2 \frac{\varphi}{2}}$$

$$BC = \frac{d\varphi}{\cos^2 \frac{\varphi}{2}}$$

2. *Equidistant projection* (Fig. 26). The image of each point projected with radius R_1 on the tangential plane laid across point A lies in its own meri-

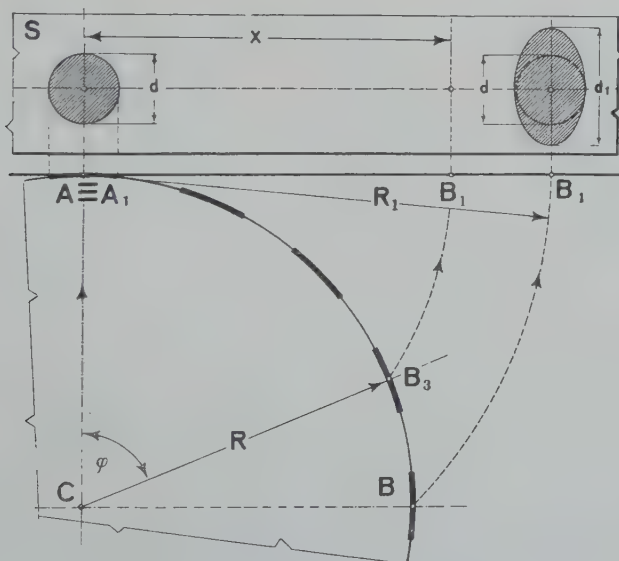


Fig. 26. The principle of equidistant projection

dional plane at a distance equal to the spherical distance of the projected point from A : $\overline{A_1 B_1} = \overline{AB}$. In this case $\overline{A_1 B_1} = R \varphi$ similarly $\overline{A_1 B_1} = R \varphi = k \operatorname{tg} \alpha$ and finally $\varphi = \frac{k}{R} \operatorname{tg} \alpha = A \operatorname{tg} \alpha$.

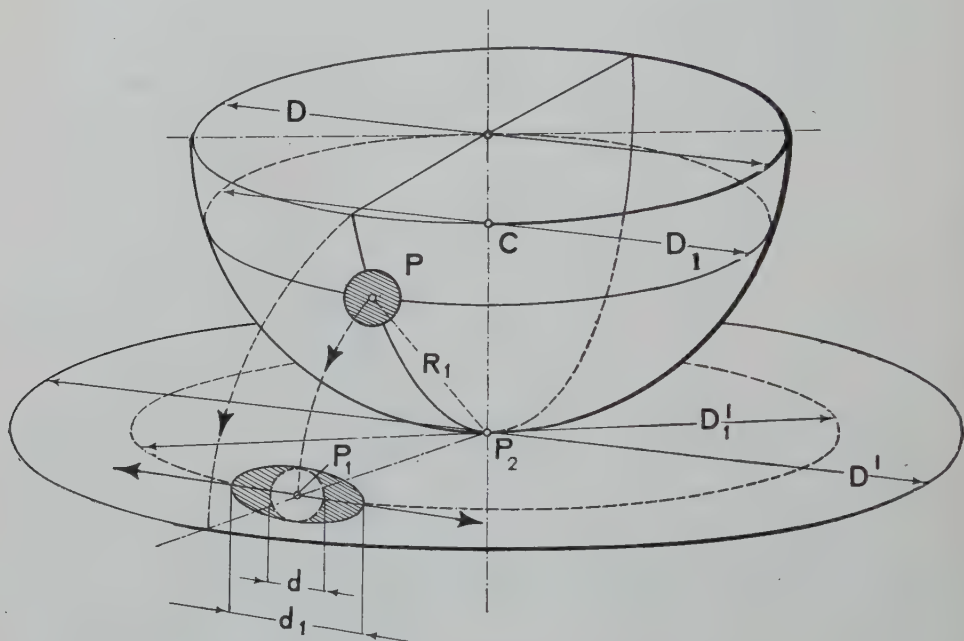


Fig. 27. Equidistant projection: the diameter of circular areas lying in the meridian remains unchanged in the one direction while increasing in the other

Diameter d or the projections of the small circles on the meridian remains unchanged in the one direction, and is increased in the other. The explanation thereof is diagrammed in Fig. 27. The sphere of diameter D is projected in the sense of the arrows upon the plane perpendicular to the vertical axis and laid across point P_2 . The projection D_1 of the circle of diameter D is larger than the original. If, now, point P at a distance R_1 on the one meridian along with the circle with diameter D_1 passing through point P is projected, its projection will be, in accordance with the above considerations, larger than the original. Thus, the projection of point P and of circle P_1 remains unchanged in the direction of one d whereas in the direction of d_1 it is extended in the sense of the arrows drawn from point P_1 .

3. *Orthographic projection* (Fig. 28). The points of the sphere are being projected perpendicular to the tangential plane A . In this case $\overline{A_1 B_1} = R \sin \varphi$. Similarly to the foregoing $\overline{A_1 B_1} = R \sin \varphi = k \operatorname{tg} \alpha$ and finally $\sin \varphi = \frac{k}{R} \operatorname{tg} \alpha =$

$= A \operatorname{tg} \alpha$, where $A = \frac{k}{R}$. The diameter of the projected images of the meridian circles increases in the one direction and decreases in the other, until the image of point A_2 , that is, of the circle, is reduced to a straight line.

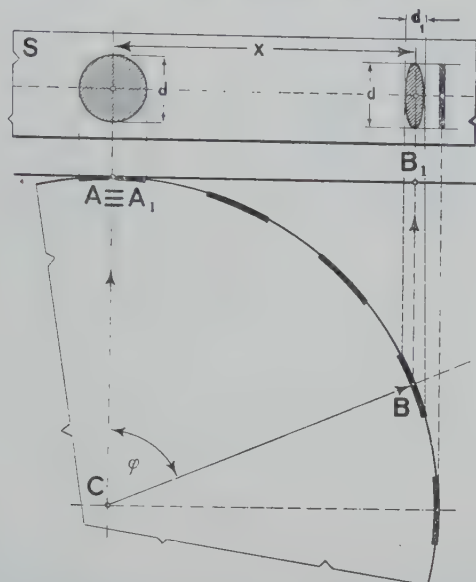


Fig. 28. The principle of orthographic projection

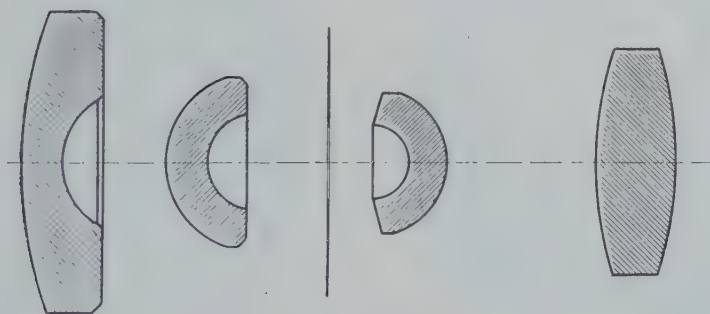


Fig. 29. Panoramic lens of Schultz

The *Hill* lens satisfies the condition set under 2, since, in case the distances on the image plane are equal, the angular distances on the hemisphere are also equal. The individual sections of the distorted image may be corrected subsequently.

Various "horizontal systems" forming panoramic images have been designed. The *Schultz* lens illustrated in Fig. 29 covers a field of $150\text{--}160^\circ$. One of the latest developments is the *Merté* lens, represented in Fig. 30.

A variety of "optical systems" similar to those described above may be encountered in the animal world. The "eyes" or rather light reception organs of most insects work according to the principle disclosed above. They comprise a group of elementary pyramidal receptors, of largely

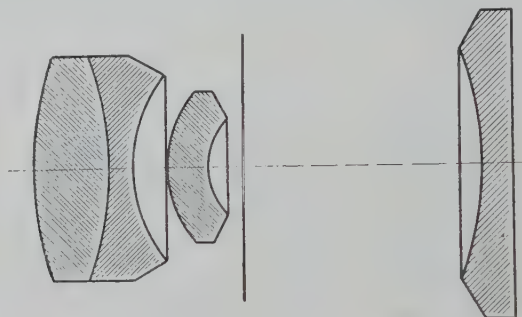
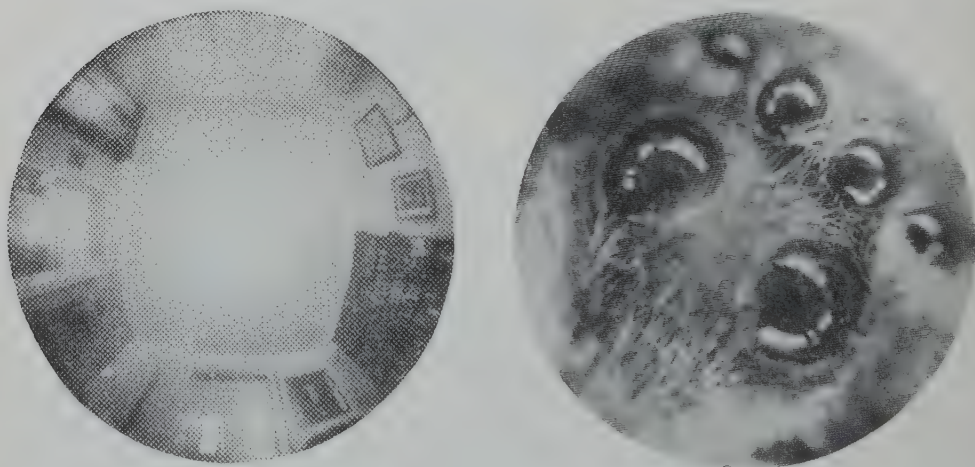


Fig. 30. Panoramic lens of *W. Merté*

hexagonal cross-sections, each of them pointing towards the interior, and optically separated from each other. Incident rays of light are received by the nerve endings at the peak of the pyramid, whereas laterally incident rays are absorbed by the pigment partitions, separating the receptors. Obviously, these arrangements cannot be considered as image-forming optical systems, they only promote the animals' orientation based on the sensation of light and shadow. For example, the dragon-fly's bigger-than-hemisphere eyes composed from elements as described



Figs. 31 and 32. The left hand figure illustrates the visual image supposed to be formed by the one eye of a May fly, the eye being composed of a group of simple eyes. This type of animal eye is, however, unsuited for image formation in the optical sense, and should be correctly called a light perception organ, as it is mainly responsive to light effects. Fig. 32. The right hand figure is a $70\times$ magnification of an image formed by the simple eyes of a cross-spider which, contrary to the May fly's eyes, are able to form "visual images". The approximately 900 diopter eye is probably highly myopic. Nature has created wide-angle lenses long before man. The technicalities of printing have, unfortunately, impaired the visibility of fine structural details of the image, but the reflected images of the window panes can, nevertheless, be well seen

above, are suited for forming overlapping images covering the entire surrounding space, except for that part of the space actually occupied by its own body. Human imagination is hardly able to follow the idea of a similar image. As far as one can speak of image formation at all, the dragonfly's one eye probably forms an image similar to the one represented in Fig. 31. Next to the intersection of the two hemispheres there are a few point-like eyes of steep curvature, perceivable also for an unaided eye. These assist the insect in tracing its prey.

Summary

It has been attempted to give a short historical summary of wide-angle photographic systems operating with discontinuous paths of ray, including the entire range from pinhole cameras to the most recent optical arrangements, from the point of view of optics as well as optomechanics. A detailed and comprehensive survey of all existing types of instruments would fall outside the scope of the present paper.

Professor DR. N. BÁRÁNY, Budapest, XI., Gombocz-Z. ú. 17

THE INFLUENCE OF THE LAYOUT AND DYNAMIC CHARACTERISTICS OF SERVOMECHANISMS ON THE TEMPERATURE CONDITIONS OF SEPARATELY EXCITED D.C. SERVMOTORS USED IN THE SERVOMECHANISMS

By

A. FRIGYES

Institute for the Theory of Operation of Electrical Machines, University of Technical Sciences, Budapest, Prof. Dr. K. P. Kovács

(Received March 30, 1957)

Nomenclature

In general the time functions are denoted with small letters, the Fourier or Laplace-transforms with capitals.

- u terminal voltage of the d. c. motor
- U transform of u
- u_0 steady-state value of the terminal voltage of the d. c. motor
- W heat generated in the armature of the motor during the transient phenomenon
- Θ inertia of the motor
- v speed of the motor
- ν transform of v
- v_∞ steady-state value of v
- m load torque of the d. c. motor
- M transform of m
- i armature current of the d. c. motor
- I transform of i
- i_∞ steady-state value of i
- T_m electromechanical time constant of the motor (See Eq. 42c)
- T_v time constant of the armature of the d. c. motor (See Eq. 42b)
- R_m resistance of the armature of the d. c. motor
- R_a additional resistance in the armature circuit
- R total resistance of the armature circuit
- Φ main flux of the d. c. motor
- K_m gain of the d. c. motor (See Eq. 42a)
- Y_m transfer function of the d. c. motor relating to the terminal voltage (See Eq. 44)
- Y_t transfer function of the d. c. motor relating to the load torque (See Eq. 45)
- Y_f transfer function of the internal feedback used in the servomechanism (See Fig. 3c)
- Y_r transfer function of the gear train (See Eq. 6)
- Y resultant transfer function of the open-loop of the servomechanism
- Y^* resultant transfer function of the closed-loop of the servomechanism for which holds

$$Y^* = \frac{Y}{1 + Y}$$

I. Introduction

In high quality servomechanisms of larger output a frequently used final control element is the separately excited direct current servomotor. The operating conditions of a servomotor used as a final control element greatly

differs from the normal operating conditions of electric motors to which the characteristic data of the motors (rated voltage, current, speed, output, losses, etc.) are related. In cases where motors rated for normal operating conditions are to be used in servomechanisms, it must be carefully examined, under what conditions the motors can be used as servomotors. Furthermore it has to be determined how a motor suitable for a certain servomechanism can be selected on the basis of its rated data.

If a motor is to be used in a servomechanism it is necessary to know in the first place the values of the time constants. Taking these into consideration the general layout of the control system should be selected and the servomechanism be designed.

In addition to this there is need of a method for giving information on the size (normal rated output) of the motor to be selected. It must be remembered that these motors are constantly in transient operation, and thus the load torque is not characteristic for the temperature conditions of the motor.

Among the losses arising in the transient operation of the servomotor only the copper losses differ considerably from the losses in steady-state conditions. The iron losses depend exclusively on the speed and the motor must not even temporarily be run at a speed considerably higher than its rated speed. Therefore the difference between the temperature rise of servomotors and that of motors in normal operation must be determined on the basis of copper losses.

It is a well-known fact that if a voltage u_0 is suddenly applied to a single running separately excited d. c. motor from a voltage source having negligible internal resistance, the amount of heat generated in the rotor is equal to the kinetic energy accumulated in the rotating parts.

$$W = \frac{1}{2} \theta v_{\infty}^2. \quad (1)$$

In expression (1) it is assumed that the inductivity L of the armature circuit is zero and the motor is not loaded with any external torque.

It can be proved that taking the inductivity L of the armature into consideration and in case of a constant load torque m , the heat arising during the starting period is

$$W = \frac{1}{2} \theta v_{\infty}^2 + \frac{1}{2} i_{\infty}^2 L + 2 m v_{\infty} T_m. \quad (2)$$

in addition to the heating power assumed constant, which causes in the armature a resistance R_m by the steady-state current i_{∞} corresponding to the external load torque m in steady-state. (See App. II.)

If thus a unit-step voltage is switched on a separately excited d. c. motor, the heat generated during the starting period consists of three parts in addition to the steady-state copper loss $i_{\infty}^2 R_m$. One part of this heat is the kinetic energy accumulated in the rotating parts, a second part is the magnetic energy of the rotor, a third part is equal to the mechanical energy consumed by the driven object during the time $2T_m$.

To estimate the order of magnitude of the individual terms of the equation, let us introduce the relative voltage drop γ connecting voltage u_0 and current i_{∞} .

$$\gamma = \frac{i_{\infty} R_m}{u_0} = \frac{i_{\infty}}{i_{rz}} = \frac{m}{m_{rz}} \quad (3)$$

where i_{rz} and m_{rz} are the short circuit (starting) current and torque respectively corresponding to voltage u_0 .

So after some simple conversions not detailed herewith, in place of Eq. (2), the following can be obtained for the heat generated during the starting period:

$$W = \frac{1}{8} \frac{u_0^2}{R_m} T_m \left[(1 - \gamma)^2 + \frac{T_v}{T_m} \gamma^2 + (4\gamma - 4\gamma^2) \right]. \quad (4)$$

The sequence of the three terms between square-brackets in the last expression corresponds to that of the terms in Eq. (2).

If the relative ohmic voltage-drop is sufficiently small, the second and third term of the expression between the square brackets are small, as compared to the first term. Therefore the heat generated is with good approximation equal to the kinetic energy accumulated in the rotor

$$\frac{1}{2} \theta v_{\infty}^2 = \frac{1}{2} \frac{u_0^2}{R_m} T_m (1 - \gamma)^2 \quad (5)$$

II. The amount of heat generated under transient conditions in the armature circuit of motors used in servomechanisms

If the d. c. motor constitutes an element of a servomechanism, the copper losses under transient conditions depend on the characteristics of the reference input. Without giving up the claim for generality, the block diagram of the control system may be assumed to be in accordance to Fig. 1. A possible feedback varying with frequency and spanning the motor (transfer element Y_m) can be incorporated in Y_v , those feedbacks, however, not spanning the motor can be converted into series elements according to the known rules of the block algebra, and can thus be incorporated in Y_e .

Y_r is the transfer function of the gear train coupled to the motor. The integrating effect of the motor can be transferred to this element by regarding the angular velocity ν as the output quantity of the motor. Thus the transfer function is

$$Y_r = \frac{X_k}{\nu} = \frac{K_r}{p}. \quad (6)$$

The d. c. motor with constant excitation is characterized as a linear element by two transfer functions. One of them describes the variation of the angular velocity with the terminal voltage, the other that with the torque.

As is known the two functions are

$$Y_m = \frac{K_m}{p^2 T_m T_r + p T_m + 1} \quad (7)$$

and

$$Y_t = \frac{-R_m K_m^2 (1 + p T_v)}{p^2 T_m T_r + p T_m + 1}. \quad (8)$$

(See App. II.)

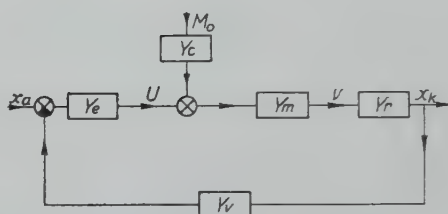


Fig. 1

Consequently for the Laplace-transforms of the speed the following can be written :

$$v = U \frac{K_m}{p^2 T_m T_r + p T_m + 1} - M \frac{R_m K_m^2 (1 + p T_v)}{p^2 T_m T_r + p T_m + 1}. \quad (9)$$

From the fundamental equations of the d. c. motor, however, the variation of the transform I of the armature current with the transform U of the terminal voltage and with the transform M of the load torque can also be determined in the following way (See App. II.) :

$$\begin{aligned} I &= \frac{U}{R_m} \frac{p T_m}{p^2 T_m T_v + p T_m + 1} - M \left(\frac{K_m p T_m (1 + p T_v)}{p^2 T_m T_v + p T_m + 1} - K_m \right) = \\ &= U \frac{p T_m}{R_m K_m} Y_m + M Y_m. \end{aligned} \quad (10)$$

If the servomotor is the element of a servomechanism, the independent variable is not the terminal voltage u but the reference input x_a . In this case the variation of the transform I of the armature current i with the transforms X_a and M of the reference input and of the torque, respectively, may be determined. Accord-

ing to the block diagram given in Fig. 1

$$U = X_a \frac{Y_e}{1 + Y_e Y_m Y_r Y_v} - M \frac{Y_t Y_m Y_r Y_v Y_e}{1 + Y_e Y_m Y_r Y_v} = \\ = X_a \frac{Y_e}{1 + Y} - M Y_t \frac{Y}{1 + Y}, \quad (11)$$

if $Y = Y_e Y_m Y_r Y_v$.

By substituting the above expression of U in the expression of I as given in Eq. (10) we have got

$$I = X_a \frac{pT_m}{R_m K_m} \cdot \frac{Y_e Y_m}{1 + Y} - M \frac{pT_m}{R_m K_m} \frac{Y_t Y_m Y}{1 + Y} + M Y_m. \quad (12)$$

In a given case knowing the time variations of the reference input and of the torques, furthermore the transfer functions, a rational fractional function is obtained for I . By inversely transforming this we obtain the time function of the current:

$$\mathcal{L}^{-1}[I(p)] = i(t).$$

This may serve as a basis for determining the heat generated in the armature of the motor.

It is known, however, that the inverse transformation cannot be carried out by purely algebraic means on account of the high order of the denominator of the reference function. There is, however, no need for it as Raleigh's theorem for Fourier transform gives a possibility to determine without inverse transformation directly from the transform the improper integral of the square of time function.

It is also known, however, that the Fourier transformation can be defined only for absolute integrable functions $f(t)$, i. e. for those, which

$$\int_{-\infty}^{+\infty} f(t) dt$$

is finite. Therefore our examination must be carried out by adapting such a reference function at which the current $i(t)$ of the motor approaches a steady-state value i_∞ . For in this case the absolute integrable function

$$f(t) = i(t) - i_\infty \quad (13)$$

of Raleigh's theorem may be applied. The current $i(t)$ approaches a finite value in case of a step or constant-velocity reference input.

In this case the heating power $P(t) = i(t)^2 R_m$ generated in the rotor is as indicated in Fig. 2 when plotted against time.

The steady-state value of $P(t)$ is

$$P_{\infty} = i_{\infty}^2 R_m.$$

As the total heat generated during the starting period the infinite range integral of the power

$$P(t) - P_{\infty}$$

will be defined. (See Fig. 2)

$$W = \int_0^{\infty} [P(t) - P_{\infty}] dt = R_m \int_0^{\infty} [i(t)^2 - i_{\infty}^2] dt. \quad (14a)$$

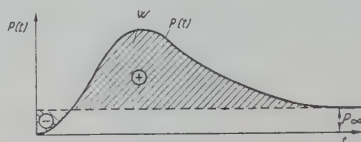


Fig. 2

Thus this is the heat generated on account of the transient phenomenon in addition to the heating power caused by the current i_{∞} corresponding to the given load.

On the basis of Eq. (14a) we shall express W in the following form:

$$W = R_m \int_0^{\infty} (i^2 - i_{\infty}^2) dt = R_m \int_0^{\infty} [(i - i_{\infty})^2 + 2i_{\infty}(i - i_{\infty})] dt. \quad (14b)$$

As W is expressed now as the integral of the function $f(t) = i - i_{\infty}$, the first term of Eq. (14b) can be determined by means of Raleigh's theorem while the second term, by means of the known final value and initial value theorem, holding for Fourier transformations.

Our further investigations will be continued in that case, where the external load of the motor is zero. For in this case $i_{\infty} = 0$, hence the second term between the square brackets is also equal to zero. This neglect gives a good approximation of the actual conditions, provided the size of the servomotor is correctly selected, in view of the required accuracy of the control.

If for some reason the term $2i_{\infty}(i - i_{\infty})$ in Eq. (14b) is required, it must be taken into account that integral of infinite range can be calculated by the method given in App. II. (See the derivation of Eq. (55).)

Let us therefore examine the servomechanism having a layout as shown in Fig. 1 to which a constant-velocity input of a slope v is applied. The speed of the reference input has to be of the value to which the rated speed of the motor corresponds in steady state.

As the transform of the reference input is

$$X_a = \frac{v}{p^2},$$

furthermore

$$M = 0,$$

therefore according to Eq. (12)

$$I = \frac{v}{p^2} \frac{p T_m}{R_m K_m} \frac{Y_e Y_m}{1 + Y}. \quad (15)$$

After some alterations, taking into consideration that $Y = Y_e Y_m Y_r Y_v$ and $Y_r = \frac{K_r}{p}$ we have

$$I = \frac{v T_m}{R_m K_m K_r} \cdot \frac{1}{Y_v} \cdot \frac{Y}{1 + Y}. \quad (16)$$

But $\frac{v}{K_r} = v_\infty$ (being at least a control system of type 1 under discussion, at which the speed of the controlled variable is also v in steady state), furthermore

$$\frac{1}{Y_v} \frac{Y}{1 + Y} = Y^*(p) \quad (17)$$

is the resultant reference function of the closed-loop. Therefore

$$I(p) = \frac{v_\infty T_m}{R_m K_m} Y^*(p). \quad (18)$$

Because a load torque $m = 0$ has been assumed, the steady-state value of the armature current is equal to zero as can be seen from Eq. (18) by means of the final value and initial value theorem. Consequently, the second term between the square-brackets in Eq. (14b) giving the heat during the transient phenomenon is zero.

As $I(p)$ expressed by Eq. (18) is the transform of an $i(t)$ which is absolute integrable, $j\omega$ can simply be substituted for p in the expression of $I(p)$, thus the Fourier-transform of the current being obtained.

$$I(j\omega) = \frac{v_\infty T_m}{R_m K_m} Y^*(j\omega). \quad (18a)$$

According to Raleigh's theorem holding for Fourier transforms, if $F(j\omega)$ is

the transform of some absolute integrable function $f(t)$, then

$$\int_{-\infty}^{+\infty} [f(t)]^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |F(j\omega)|^2 d\omega. \quad (19)$$

Accordingly, the total heat generated in the armature is

$$\begin{aligned} W &= R_m \int_0^{\infty} [i(t)]^2 dt = \frac{R_m}{2\pi} \int_{-\infty}^{+\infty} |I(j\omega)|^2 d\omega = \\ &= R_m \frac{v_{\infty}^2 T_{\infty}^2}{R_m^2 K_m^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y^*(j\omega)|^2 d\omega. \end{aligned} \quad (20)$$

If it is considered that the electromechanical time constant in the last expression is

$$T_m = \theta R_m K_m^2,$$

then after some simple conversion we obtain

$$W = v_{\infty}^2 \theta T_m \frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y^*(j\omega)|^2 d\omega. \quad (21)$$

For computing improper integrals of such a form known formulae can be found in the literature provided, $Y^*(j\omega)$ is a rational fractional function the numerator of which is of lower order than the denominator.

In App. I/a the value of the integrals

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y^*(j\omega)|^2 d\omega$$

is indicated. In our case for $Y^*(p)$ which is nothing else but the transfer function of the closed-loop, the following can be stated :

1. $Y(0) = 1$.
2. The order of the numerator is lower by at least two, than that of the denominator.

The statement 1 concludes from the type number of the control system. As for statement 2 derives from the fact that the servomotor and the succeeding gear train constitute a series element of the system, in the resultant transfer function of which the numerator is of an order lower of at least two, than the denominator. As for the transfer function of the remaining elements of the system, the order of its numerator is not higher than that of its denominator.

Consequently, in our case in the expressions given in App. I/a $B_{n-1} = 0$ and $A_0 = B_0$. For numerical calculations these expressions can be converted into simpler forms. To this end the resultant gain K of the closed-loop, the value of which can be proved to be 1 in case of a system of type 1 (See App. III), will be lifted out of the expressions.

$$K = \frac{A_0}{(A_1 - B_1)} \quad (22)$$

Thus the mentioned expressions can be converted into the following form :

$$J_k = \frac{K}{2} F_k = \frac{A_0}{2(A_1 - B_1)} F_k. \quad (23)$$

Hence the total heat generated in the armature circuit

$$W = \frac{\theta v_\infty^2}{2} T_m K F_k. \quad (24)$$

The values of F_k are given in App. I/b.

As an example, the heat generated in the armature circuit has been determined for the case of servomotors, as according to Figs. 3a, 3b, 3c, provided a constant-velocity reference input is applied. The numerical values chosen are indicated in the figures. The values of $T_m K F_k$ are given in the following table.

$T_m = 0,25 \text{ sec}$				
	K	F_k	$T_m K F_k$	W
2a	10/sec	2,02	3,03	$3,03 \frac{\theta v_\infty^2}{2}$
2b	30/sec	1,61	7,26	$7,26 \frac{\theta v_\infty^2}{2}$
2c	30/sec	1,88	8,46	$8,46 \frac{\theta v_\infty^2}{2}$

Hence it can be seen that the heat generated during the transient phenomenon is in case of a) appr. 3 times, in case of b) appr. 7,3 times, in case of c) appr. 8,5 times, as much as, the kinetic energy accumulated in the rotating parts.

The heat calculated above is generated in the entire armature circuit, thus in case of a motor being supplied from an amplidyne, this is the total heat generated in the armature of the motor, plus in that of the amplidyne. This heat is evidently divided between the motor and the amplidyne in the ratio of their resistances.

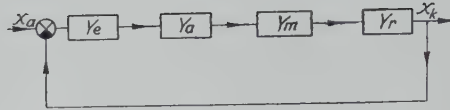


Fig. 3/a

$$Y_e = K_e; Y_m = \frac{K_m}{1 + p T_m}; Y_a = \frac{K_a}{(1 + p T_{a1})(1 + p T_{a2})}; Y_r = \frac{K_r}{p}$$

$$K_e = 0,1; K_a = 10; K_m = 2 \frac{1}{\text{Vsec}}; K_r = 5 \text{ V}$$

$$T_{a1} = 0,03 \text{ sec}; T_m = 0,15 \text{ sec}; T_{a2} = 0,03 \text{ sec};$$

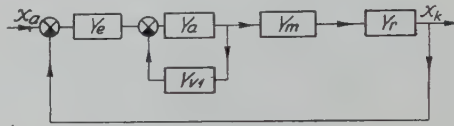


Fig. 3/b

$$Y_{v1} = K_{v1}$$

$$K_e = 0,6; K_a = 10; K_m = 2 \frac{1}{\text{Vsec}}; K_r = 15 \text{ V}; K_{v1} = 0,5$$

$$T_{a1} = 0,03 \text{ sec}; T_m = 0,15 \text{ sec}; T_{a2} = 0,03 \text{ sec};$$

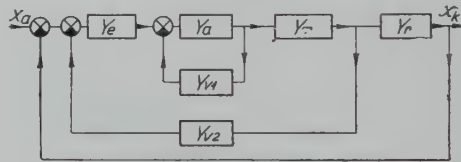


Fig. 3/c

$$Y_{v2} = K_{v2}$$

$$K_e = 1,8; K_a = 10; K_m = 2 \frac{1}{\text{Vsec}}; K_r = 20 \text{ V}; K_{v1} = 0,5; K_{v2} = 0,5$$

$$T_{a1} = 0,03 \text{ sec}; T_{a2} = 0,03 \text{ sec}; T_m = 0,15 \text{ sec}$$

III. The determination of the optimum transfer function in view of the temperature conditions of the motor

By means of the expression derived above, the copper loss in the servomotor of a servomechanism designed according to specified control qualities (i. e. response time, overshoot, etc.) can be determined when following-up a unit constant-velocity reference input. It can be of interest to determine the layout at which the amount of heat will be at minimum.

Accordingly, we could proceed by determining the minimum value of F_k while varying some parameters according to the conventional method for computing extreme values. Unfortunately, by this procedure such a value of

the parameter concerned is obtained, that cannot be realized either technically, or at which the control quality is unacceptable.

Therefore another approach should be made for the solution of the problem. The values characteristic for the quality of the control (response time, overshoots, etc.) should be specified, and an output signal should be sought for, by which these requirements are just satisfactory, the copper loss in the rotor of the servomotor being at the same time a minimum. Such transfer elements at which this ideal output signal would arise cannot be realized in practice. If, however, the amount of heat is determined that could be generated in this

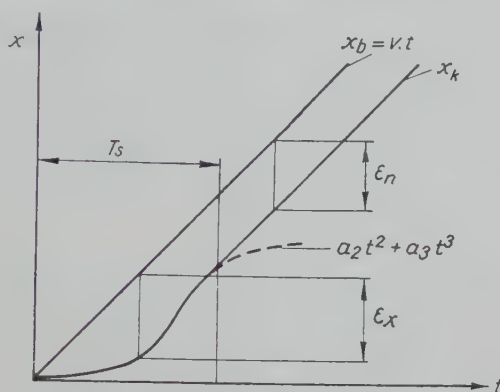


Fig. 4

ideal case, this gives a good basis of comparison to decide, whether for the sake of improving the temperature conditions of the motor is it advisable to make alterations in the layout or parameters of the control system selected in view of the quality of the control, and to see at all whether considerably better results could be obtained. If not, the parameters selected on the basis of the quality of the control should be retained.

A) The analytical expression of the optimum output signal

The current of the idly running motor is proportional to the acceleration of the output signal. Because

$$i = K_m \theta \frac{d v}{d t} = \frac{K_m \theta}{K_r} \frac{d^2 x_k}{d t^2},$$

where K_r is the transfer coefficient of the gear train succeeding the motor. Our task is to determine such a

$$x_k = x_k(t)$$

function for which the integral

$$W = \int_0^{\infty} i^2(t) R_m dt = \frac{R_m \Theta^2 K_m^2}{K_r^2} \int_0^{\infty} \left(\frac{d^2 x_k}{dt^2} \right)^2 dt \quad (25)$$

is a minimum.

It can be assumed that the follow-up of such an input is dealt with for which the steady-state value of the current is zero (e. g. in case of a unit-step or a unit constant-velocity input), hence from the output signal it can be assumed to attain its steady-state value or its derivative in time T_s unknown for the present, while from there it can be substituted by a straight line. (See Fig. 4) As for $t > T_s$ the acceleration is zero, therefore in the above expression the limit of integration is t_s instead of ∞ . Under these circumstances our task is actually the solution of a variation calculus problem.

We shall apply the *Euler-Poisson* equation according to which the function

$$y = y(x)$$

for which the integral

$$H = \int_{x_2}^{x_1} F \left(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2} \right) dx \quad (26)$$

is a minimum, can be expressed by the following differential equation :

$$\frac{dF}{dy} - \frac{d}{dx} \frac{dF}{dy'} + \frac{d^2}{dx^2} \frac{dF}{dy''} = 0. \quad (27)$$

Here

$$y' = \frac{dy}{dx} \quad \text{and} \quad y'' = \frac{d^2 y}{dx^2}.$$

In our case

$$\begin{aligned} x &= t \\ y &= x_k, \quad y' = \frac{dx_k}{dt}, \quad y'' = \frac{d^2 x_k}{dt^2} \\ H &= W \end{aligned}$$

and

$$F = \frac{R_m \Theta^2 K_m^2}{K_r^2} \left(\frac{d^2 x_k}{dt^2} \right)^2.$$

Because in our case F depends only on its second order derivative, therefore

$$\frac{dF}{dy} = 0,$$

$$\frac{dF}{dy'} = 0,$$

$$\frac{dF}{dy''} = \frac{R_m \theta^2 K_m^2}{K_r^2} \cdot 2 \frac{d^2 x_k}{dt^2}. \quad (27)$$

By substituting these values into the above differential equations we get

$$\frac{d^2}{dt^2} \cdot \frac{R_m \theta^2 K_m^2}{K_r^2} \cdot 2 \frac{d^2 x_k}{dt^2} = 0 \quad (28)$$

that is

$$\frac{d^4 x_k}{dt^4} = 0. \quad (29)$$

The general solution of this differential equation is as follows :

$$x_k = a_3 t^3 + a_2 t^2 + a_1 t + a_0. \quad (30)$$

The constants in this differential equation can be determined by taking the following boundary conditions as a basis (See Fig. 4) :

1	if $t = 0$	$x_k = 0$
2	if $t = 0$	$\frac{dx_k}{dt} = 0$
3	if $t = T_s$	$x_k = v T_s - \varepsilon_n$
4	if $t = T_s$	$\frac{dx_k}{dt} = v.$

The first two conditions express that at the first instant the output signal as well as its derivative are equal to zero, the latter being so, on account of the inertia of the motor. The conditions 3 and 4 express that the output signal continuously and with a continuous tangent passes through to the steady state substituted with the straight line.

Based on these conditions the following expressions are obtained for the values of the constants :

$$a_0 = 0,$$

$$a_1 = 0,$$

$$a_2 = \frac{2 v T_s - 3 \varepsilon_n}{T_s^2} = \frac{v}{K T_s^2} (2 K T_s - 3) \quad (31)$$

$$a_3 = \frac{2 \varepsilon_n - v T_s}{T_s^3} = \frac{v}{K T_s^3} (2 - K T_s). \quad (32)$$

In the expressions of a_2 and a_3 it is implied that a control system of the type 1 is under discussion, therefore the following relation holds true between the velocity of the reference input and the steady-state error :

$$\frac{v}{\varepsilon_n} = K$$

where K is the gain of the open-loop.

The ideal output signal chosen, according to the mentioned above is shown in Fig. 4. It can be well seen that the error has a maximum value. Let us denote the quotient of this maximum and the steady-state error with γ .

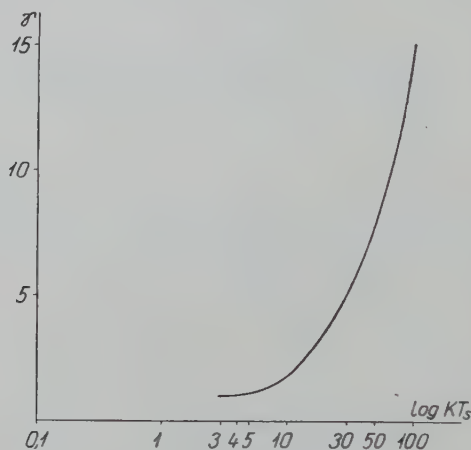


Fig. 5

It can be verified in a simple way (See App. IV) that the value of the quotient is

$$\frac{\varepsilon_x}{\varepsilon_s} = \gamma = \frac{(KT_s)^2 (4KT_s - 9)}{27(KT_s - 2)^2} \quad (33)$$

The relation between γ and KT_s as given by Eq. (33) is shown in Fig. 5.

B) The heat generated in case of the optimum output signal

In order to determine the ideal output signal suitable for a given control task, beside the gain K either the response time T_s or the value of γ characteristic for the maximum dynamic error must be accessed beforehand.

As according to the above discussion, the current of the servomotor is proportional to the second order derivate of the output signal, hence by dif-

differentiating Eq. (30) twice, and by substituting it into Eq. (25), for the total heat generated during the starting period, in this ideal case

$$W = \frac{R_m \Theta^2 K_m^2}{K_r^2} \int_0^{T_s} (2a_2 + 6a_3 t)^2 dt$$

is obtained. By carrying out the integration and substituting the values of the constants from Eqs. (31) and (32), furthermore taking into consideration that

$$\frac{v}{K_r} = v_\infty$$

and

$$\theta R_m K_m^2 = T_m$$

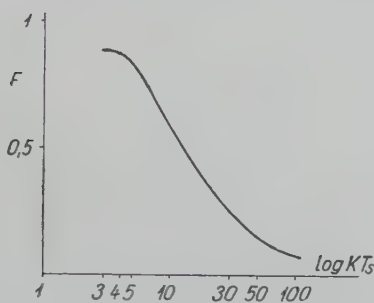


Fig. 6

we have

$$W = \frac{\theta v_\infty^2}{2} \frac{T_m}{T_s} \cdot \frac{8 [(KT_s)^2 - 3KT_s + 3]}{(KT_s)^2}. \quad (34)$$

In order to make a comparison possible of the heat generated in case of the practicable control system with that calculated above, it is worth while to carry out the following simple alteration:

$$\begin{aligned} W &= \frac{\theta v_\infty^2}{2} T_m K \cdot \frac{8 [(KT_s)^2 - 3KT_s + 3]}{(KT_s)^3} = \\ &= \frac{\theta v_\infty^2}{2} T_m K \cdot F_i \end{aligned} \quad (35)$$

where

$$F_i = 8 \frac{(KT_s)^2 - 3KT_s + 3}{(KT_s)^3}. \quad (36)$$

In Fig. 6 the variation of F_i with KT_s as expressed by the above equation is shown. By comparing this with Fig. 5, F_i can be plotted against γ . (See

Fig. 7) From this it can be seen that the maximum value of F_i is $8/9$ for $\gamma = 1$. For $\gamma = 1,5$, $F_i = 0,67$.

Taken into account the afore-mentioned it can be stated that in case of an output signal that is optimum concerning the temperature rise of the motor, and in case a control system advantageous as for overshoots, the value of F_i may be chosen to be between $0,7 \sim 0,9$. Therefore the minimum amount of heat generated in the armature of the servomotor is

$$W = 0,7 \sim 0,9 \cdot T_m K \frac{\theta \gamma_{\infty}^2}{2}.$$

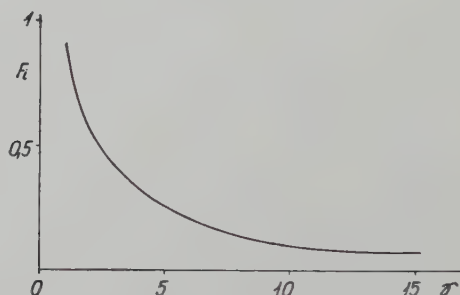


Fig. 7

It is evident that this cannot be achieved in practice, because such transfer elements are not available, by means of which the ideal output signal could be produced. Calculations carried out in several cases show that in practice the value of F_i can be reduced to appr. 1,4.

The calculation procedure shown above can be used not only to determine the temperature conditions of servomotors, being an element in a closed-loop, but it can also be used to examine the temperature conditions of motors in intermittent operation, the armatures of which are supplied through elements varying with frequency. This is true e. g. for Ward-Leonard drives of intermittent operation and of those controlled by amplidynes.

Appendix I/a

$$Y^*(p) = \frac{B_{n-1} p^{n-1} + \dots + B_0}{A_n p^n + \dots + A_0} *$$

$$J = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y^*(j\omega)|^2 d\omega$$

*James—Nichols—Phillips: Theory of Servomechanisms. McGraw-Hill Book Company, Inc. 1947.

$$J_1 = \frac{B_0^2}{2A_0 A_1}$$

$$J_2 = \frac{B_1^2 + \frac{A_2 B_0^2}{A_0}}{2 A_1 A_2}$$

$$J_3 = \frac{A_1 B_2^2 + A_3 B_1^2 - 2 A_3 B_0 B_2 + \frac{A_2 A_3 B_0^2}{A_0}}{2 A_3 (A_1 A_2 - A_0 A_3)}$$

$$J_4 = \frac{B_3(A_1 A_2 - A_0 A_3) + B_3^2 A_1 A_4 - 2 B_1 B_3 A_1 A_4 - B_1^2 A_3 A_4 - 2 B_0 B_2 A_3 A_4 + \frac{A_4 B_0^2}{A_0} (A_2 A_3 - A_1 A_4)}{2 A_4 (A_1 A_2 A_3 - A_1^2 A_4 - A_0 A_3^2)}$$

Appendix I/b

$$F_1 = 1,$$

$$F_2 = 1,$$

$$F_3 = \frac{\frac{B_1^2}{A_0} + A_2}{\frac{A_1 A_2 - A_0 A_3}{A_1 - B_1}}$$

$$F_4 = \frac{B_2^2 \frac{A_1}{A_0} - B_1^2 \frac{A_3}{A_0} - 2 B_2 A_3 + (A_2 A_3 - A_1 A_4)}{\frac{A_1}{A_1 - B_1} (A_2 A_3 - A_1 A_4) - \frac{A_0 A_3^2}{A_1 - B_1}}$$

$$F_5 = \frac{B_3^2 \frac{A_{12}}{A_0} + (B_3^2 - 2 B_1 B_3) \frac{A_{14}}{A_0} + (B_1^2 - 2 B_0 B_2) \frac{A_{34}}{A_0} + A_2 A_{34} - A_4 A_{14}}{\frac{A_1}{A_1 - B_1} A_2 A_{34} - A_4 A_{14} + \frac{A_3}{A_1 - B_1} A_{34} - \frac{A_5}{A_1 - B_1} A_{14}}$$

where the following substitutions were carried out in F_5 :

$$A_{12} = A_1 A_2 - A_0 A_3,$$

$$A_{14} = A_1 A_4 - A_0 A_5,$$

$$A_{34} = A_3 A_4 - A_2 A_5.$$

Appendix II

The voltage equation of the d. c. motor is

$$u = R_m i + L_m \frac{di}{dt} + c \Phi v \quad (37)$$

and its torque equation is

$$c \Phi i = m + \Theta \frac{dv}{dt}. \quad (38)$$

By rewriting Eqs. (37) and (38) into Laplace-transforms, with initial value equal to zero we have

$$U = (R_m + pL_m) I + c \Phi \nu \quad (39)$$

$$c \Phi I = M + p \Theta \nu \quad (40)$$

By eliminating I from the last two equations and solving them for ν , the transfer function of the motor is obtained as follows:

$$\nu = U \frac{c \Phi}{p \Theta (R_m + pL_m) + c^2 \Phi^2} - M \frac{R_m + pL_m}{p \Theta (R_m + pL_m) + c^2 \Phi^2} \quad (41)$$

If in the above equation the following substitutions are carried out

$$\frac{1}{c \Phi} = K_m \quad (42a)$$

$$\frac{L_m}{R_m} = T_v, \quad (42b)$$

$$\frac{\Theta R_m}{c^2 \Phi^2} = \Theta R_m K_m^2 = T_m, \quad (42c)$$

the equation assumes the following form:

$$\nu = U \frac{K_m}{p^2 T_m T_v + p T_m + 1} - M \frac{R_m K_m^2 (1 + p T_v)}{p^2 T_m T_v + p T_m + 1} = U Y_m + M Y_t Y_m \quad (43)$$

where

$$Y_m = \frac{K_m}{p^2 T_m T_v + p T_m + 1} \quad (44)$$

$$Y_t = -R K_m (1 + p T_v). \quad (45)$$

On the basis of the above written the block diagram given in Fig. 8 can be substituted for the motor as a transfer element.

Now the transfer functions for current I will also be determined. To this end let us express I from Eq. (40):

$$I = \frac{M}{c \Phi} + \frac{p \Theta}{c \Phi} \nu = K_m M + p \frac{\Theta R_m}{c^2 \Phi^2} \frac{c \Phi}{R_m} \nu = K_m M + \frac{p T_m}{K_m R_m} \nu \quad (46)$$

Then substituting ν from Eq. (41) we have

$$\begin{aligned} I &= \frac{U}{R_m} \frac{p T_m}{p^2 T_m T_v + p T_m + 1} - M \left(\frac{K_m p T_m (1 + p T_v)}{p^2 T_m T_v + p T_m + 1} - K_m \right) = \\ &= U \frac{p T_m}{R_m K_m} Y_m + M Y_m. \end{aligned} \quad (47)$$

Based on these, the time variation of the velocity, and that of the current of the motor can be determined. Both have two components: one of them varies with the voltage, the other with the current. While deriving the expressions we have had to assume that the linear relations expressed by the initial equations (37) and (38) actually exist between the individual variables, furthermore the constants therein are truly constants (this refers in the first place to Φ and L_m), and the voltage u and load torque m are known variables, varying exclusively with time. This

latter conditions must particularly be emphasized in case of the torque m as e. g. the bearing friction or the windage loss depending on the speed, and the torque caused by the eddy-currents cannot evidently be taken into consideration in this expression.

Note : If a damping torque $m_d = k v$ varies linearly with the speed exists, for the functions relating to the angular velocity and to the current, it can be proved similarly to the above stated that instead of Eqs. (44), (43), (47) the following expressions hold good :

$$Y'_m = \frac{K_m}{p^2 T_m T_v + p (\alpha T_v + T_m) + 1 + \alpha}$$
$$v = U Y'_m + M Y_l Y'_m \tag{48}$$

$$I = U \frac{p T_m + \alpha}{R_m K_m} Y'_m + M Y'_m$$

where

$$\alpha = \frac{k R_m}{c^2 \Phi^2} . \tag{49}$$

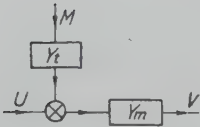


Fig. 8

In our further discussion those resistances, that are proportional to the speed will not be taken into consideration. In place of the reactive frictional torques, however, we shall assume for the time being, an active external torque.

As the constant torque requires a steady-state current $i_\infty = I_0$ to be maintained, therefore, the heat generated during the switching-on period will be defined as the following integral : (See Fig. 2)

$$W = \int_0^\infty (i^2 R_m - i_\infty^2 R_m) dt . \tag{50}$$

Let us express W as follows :

$$W = R_m \int_0^\infty (i^2 - i_\infty^2) dt = R_m \int_0^\infty [(i - i_\infty)^2 + 2i_\infty (i - i_\infty)] dt . \tag{51}$$

To determine this Raleigh's theorem will be used according to which if $F(j\omega)$ is the Fourier-transform of some absolute integrable function $f(t)$, then

$$\int_{-\infty}^{+\infty} f(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |F(j\omega)|^2 d\omega . \tag{52}$$

According to the final value and initial value theorem it follows from Eq. (47) that as a result of a unit-step change of U and M

$$i_\infty = m_0 K_m$$

(where m_0 is the value of the applied torque).

Consequently, by substituting $U = \frac{u_0}{p}$ and $M = \frac{m_0}{p}$ we have

$$\begin{aligned} L[i - i_\infty] &= u_0 \frac{T_m}{R_m K_m} Y_m + \frac{m_0}{p} Y_m - \frac{m_0 K_m}{p} = \\ &= \frac{u_0}{R_m} \frac{T_m}{p^2 T_m T_v + p T_m + 1} - m_0 K_m \frac{T_m (1 + p T_v)}{p^2 T_m T_v + p T_m + 1} = \\ &= \frac{T_m \left[\frac{u_0}{R_m} - m_0 K_m (1 + p T_v) \right]}{p^2 T_m T_v + p T_m + 1}. \end{aligned} \quad (53)$$

By substituting $j\omega$ for p in Eq. (53), we obtain the Fourier-transform of $i - i_\infty$. And to this the integral (52) is to be applied. That is

$$\int_0^\infty (i - i_\infty)^2 dt = \frac{T_m^2}{2\pi R_m^2} \int_{-\infty}^{+\infty} \left| \frac{-m_0 K_m R_m T_v j\omega + (u_0 - m_0 K_m R_m)}{(j\omega)^2 T_m T_v + j\omega T_m + 1} \right|^2 d\omega.$$

For the determination of improper integrals of such a form closed expressions are available. (See App. I/a.)

In our case using the symbole given in App. I/a

$$B_1 = -m_0 K_m R_m T_v = -i_\infty R_m T_v,$$

$$B_0 = u_0 - m_0 K_m R_m = u_0 - i_\infty R_m,$$

$$A_2 = T_m T_v,$$

$$A_1 = T_m,$$

$$A_0 = 1.$$

Hence

$$\int_0^\infty (i - i_\infty)^2 dt = \frac{T_m^2}{R_m^2} \frac{i_\infty^2 R_m^2 T_v^2 + T_m T_v (u_0^2 - 2u_0 i_\infty R_m + i_\infty^2 R_m^2)}{2 T_m^2 T_v}.$$

By multiplying the last equation by R_m , after some simple conversions we obtain

$$R_m \int_0^\infty (i - i_\infty)^2 dt = \frac{1}{2} \frac{u_0^2}{R_m} T_m + \frac{1}{2} i_\infty^2 R_m T_v - u_0 i_\infty T_m + \frac{1}{2} i_\infty^2 R_m T_m. \quad (54)$$

By means of the final value and initial value theorem, the second term between the square-brackets in Eq. (51) can very easily be integrated.

$$\int_0^\infty (i - i_\infty) dt = \left[\int (i - i_\infty) dt \right]_\infty - \left[\int (i - i_\infty) dt \right]_0.$$

But according to the final value and initial value theorem,

$$\left[\int (i - i_{\infty}) dt \right]_{\infty} = \left[\frac{1}{p} \mathcal{L}(i - i_{\infty}) \cdot p \right]_0 = [\mathcal{L}(i - i_{\infty})]_0$$

and similarly

$$\left[\int (i - i_{\infty}) dt \right]_0 = [\mathcal{L}(i - i_{\infty})]_{\infty}.$$

Making use of the Laplace-transform of $(i - i_{\infty})$ as written in Eq. (53) we have

$$\int_0^{\infty} (i - i_{\infty}) dt = T_m \frac{u_0}{R_m} - m_0 K_m T_m = T_m \frac{u_0}{R_m} = i_{\infty} T_m.$$

Hence after simple alteration we obtain

$$2 i_{\infty} R_m \int_0^{\infty} (i - i_{\infty}) dt = 2 u_0 i_{\infty} T_m - 2 i_{\infty}^2 R_m T_m. \quad (55)$$

Let us substitute the expressions given in Eqs. (54) and (55) into Eq. (51):

$$W = \frac{1}{2} \frac{u_0^2}{R_m} T_m + \frac{1}{2} i_{\infty}^2 R_m T_v + u_0 i_{\infty} T_m - \frac{3}{2} i_{\infty}^2 R_m T_m. \quad (56)$$

Let us convert the above expression in the following way:

As according to Eq. (41) the steady-state speed is

$$v_{\infty} = u_0 K_m - m_0 R_m K_m^2,$$

or

$$u_0 = \frac{v_{\infty}}{K_m} + m_0 K_m R_m = \frac{v_{\infty}}{K_m} + i_{\infty} R_m$$

therefore the first term of Eq. (56) is

$$\frac{1}{2} \frac{u_0^2}{R_m} T_m = \frac{1}{2} u_0^2 \Theta K_m^2 = \frac{1}{2} \Theta v_{\infty}^2 + m_0 v_{\infty} T_m + \frac{1}{2} i_{\infty}^2 R_m T_m. \quad (56a)$$

For the second term

$$\frac{1}{2} i_{\infty}^2 R_m T_v = \frac{1}{2} i_{\infty}^2 L m. \quad (56b)$$

In the third term the power $u_0 i_{\infty}$ consumed by the supply network in steady state can be written as the sum of the mechanical output $m_0 v_{\infty}$ and of copper loss $i_{\infty}^2 R_m$:

$$u_0 i_{\infty} T_m = m_0 v_{\infty} T_m + i_{\infty}^2 R_m T_m. \quad (56c)$$

By substituting Eqs. (56a), (56b), (56c) into Eq. (56) the total heat generated during the starting period is obtained:

$$W = \frac{1}{2} \Theta v_{\infty}^2 + \frac{1}{2} i_{\infty}^2 L + 2 m_0 v_{\infty} T_m. \quad (57)$$

Appendix III

The verity of Eq. (22) can be seen in the following way :

If the transform of the error of the servomechanism is denoted E , the error transfer function of the system is

$$\begin{aligned} \frac{E}{X_n} &= Y_\varepsilon^*(p) = 1 - Y^*(p) = 1 - \frac{B_{n-1}p^{n-1} + \dots + B_1p + B_0}{A_n p^n + \dots + A_1p + A_0} = \\ &= \frac{A_n p^n + (A_{n-1} - B_{n-1})p^{n-1} + \dots + (A_1 - B_1)p + B_0 - A_0}{A_n p^n + \dots + A_1p + A_0}. \end{aligned}$$

Considering that on account of $Y_\varepsilon^*(0) = 0$ holds $B_0 - A_0 = 0$, the transform of the error caused by unit constant-velocity input is

$$E(p) = \frac{1}{p^2} \frac{p [A_n p^{n-1} + (A_{n-1} - B_{n-1})p^{n-2} + \dots + (A_1 - B_1)]}{A_n p^n + \dots + A_1p + A_0}.$$

The steady-state value ε_∞ of the error can on one hand be computed from the above expression by means of the final value and initial value theorem, on the other hand, however, it is known that its reciprocal is equal to the reciprocal of the resultant gain in case of a system type 1. Therefore

$$\varepsilon_\infty = [p E(p)]_{p=0} = \frac{A_1 - B_1}{A_0} = \frac{1}{K}.$$

Appendix IV

Let us introduce the expression $\alpha = \frac{1}{K T_s}$. With this Eqs. (31) and (32) assume the following forms :

$$a_2 = \frac{v}{T_s} (2 - 3\alpha) \quad (58a)$$

$$a_3 = \frac{v}{T_s^2} (2\alpha - 1). \quad (58b)$$

According to Fig. 4 and Eq. (30) the error of the control is

$$\varepsilon = vt - a_3 t^3 - a_2 t^2. \quad (59)$$

The extreme value of the error can be calculated from the following expression :

$$\begin{aligned} \frac{d\varepsilon}{dt} &= v - 3a_3 t^2 - 2a_2 t = 0 \\ t_{12} &= \frac{a_2 \pm \sqrt{a_2^2 + 3a_3 v}}{-3a_3} = \\ &= \frac{\frac{v}{T_s} (2 - 3\alpha) \pm \sqrt{\frac{v^2}{T_s^2} (4 - 12\alpha + 9\alpha^2) + \frac{v^2}{T_s^2} (6\alpha - 3)}}{3 \frac{v}{T_s^2} (1 - 2)} = \\ &= T_s \frac{2 - 3\alpha \pm (3\alpha - 1)}{3(1 - 2\alpha)}. \end{aligned}$$

Hence

$$t_1 = T_s \frac{1}{3(1-2\alpha)} \quad (60a)$$

$$t_2 = T_s \quad (60b)$$

Let us build the second-order derivative of Eq. (59)

$$\frac{d^2 \varepsilon}{dt^2} = -2a_2 - 6a_3 t$$

the value of which for $t = T_s$ is

$$\left[\frac{d^2 \varepsilon}{dt^2} \right]_{t=T_s} = \frac{v}{T_s} (2-6\alpha),$$

This is positive, in other words the error has a minimum value for T_s if

$$\alpha < \frac{1}{3}.$$

In this case according to Eq. (60a) the error has its maximum value at

$$t_1 < T_s.$$

Consequently, only those curves can be of interest to us for which α is smaller than $1/3$. Let us compute the quotient γ of the maximum and minimum value of the error.

The maximum error ε_x according to Eq. (59) is

$$\begin{aligned} \varepsilon_x = v t_1 - a_3 t_1^3 - a_2 t_1^2 &= v T_s \frac{1}{3(1-2\alpha)} - \frac{v}{T_s} (2\alpha-1) \frac{T_s^3}{27(1-2\alpha)^3} - \\ &- \frac{v}{T_s} (2-3\alpha) \frac{T_s^2}{9(1-2\alpha)^2} = v T_s \frac{4-9\alpha}{27(1-2\alpha)^2}. \end{aligned}$$

As, however, $v = \varepsilon_n K$, (ε_n is the minimum value of the error, and in this case the steady-state value of it, too) is therefore

$$\varepsilon_x = \varepsilon_n K T_s \frac{4-9KT_s}{27 \left(1 - \frac{KT_s}{2}\right)^2} = \varepsilon_n (KT_s)^2 \frac{4KT_s-9}{27(KT_s-2)^2}$$

furthermore

$$\gamma = \frac{\varepsilon_x}{\varepsilon_n} = (KT_s)^2 \frac{4KT_s-9}{27(KT_s-2)^2}.$$

Summary

The article shows how to give an explicit form to the improper integral of the square current of a d. c. servomotor used in automatic feedback control systems. Thus the heat generated in the armature circuit of the servomotor can be determined.

This method of variation calculus is suitable for determining the least quantity of heat arising in the armature circuit in case of given parameters describing the behaviour of the control system. This optimal value existing among the ideal circumstances gives a good basis for considering, whether it is worth to change the layout of the realized control for reducing the quantity of heat arising in the armature circuit.

A. FRIGYES, Budapest, XI., Budafoki út 4-6, Hungary.

SHORT-CIRCUIT CURRENTS IN CIRCUITS CONTAINING SERIES CAPACITORS

By

F. CSÁKI

Department of Special Electric Machines, Polytechnical University, Budapest

(Received March 31, 1957)

The development of short-circuit currents in circuits containing a series capacitor has scarcely been dealt with in the literature. This may be ascribed to the fact that the protection of the capacitor is, for economical reasons, generally considered inevitable [1]. As the spark-gap protective device by-passes the capacitor, already during the first semi-period of the fault-current, the series capacitor has apparently been supposed to have but a small effect, if any, upon the short-circuit current.

Nevertheless, according to other authors [2] the spark gap may in some instances be omitted. In this case the capacitor obviously remains in the circuit during short circuit and will increase the short-circuit current. The question arises: to what extent? Does this endanger the transmission line, transformer, circuit breaker, etc.? On the other hand, an overvoltage rises even on the capacitor during short circuit. To what extent should the capacitor be oversized in order to endure the overvoltage? The present paper is devoted to these questions.

1. Simplifying assumptions

To simplify computation we make some assumptions adopted in practice. The magnetizing current and the iron loss of the transformers shall be neglected, and the transformer substituted by a series resistance R_{tr} and an inductivity L_{tr} . The capacitance and leakance of the transmission line are also neglected and the transmission line substituted by a series resistance R_l and an inductivity L_l . A pure capacity C , will be substituted for the series capacitor and the losses of the latter neglected. An "infinite" bus is assumed before the feed-side transformer. Consequently, during the whole period of short circuit the voltage of the feed-point shall be of steady value and frequency. We start by examining a three-phase short circuit arising at no-load.

2. Calculation of the short-circuit current

With the assumptions made, the short-circuit loop may be substituted by a simple R, L, C circuit in series, where $R = R_t + \Sigma R_{tr}$ and $L = L_t + \Sigma L_{tr}$. The short circuit is equivalent to the sudden connecting of an alternating voltage (Fig. 1).

For zero starting conditions the integral-differential equation of the circuit with *complex quantities* is :

$$L \frac{d\bar{I}}{dt} + R\bar{I} + \frac{1}{C} \int_0^t \bar{I} dt = \bar{U} = U_m e^{j(\omega t + \psi)} \quad (1)$$

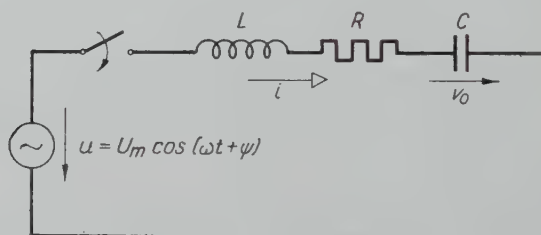


Fig. 1

By Laplace transformation* the operational function of the current may be easily computed :

$$\bar{I}(p) = U_m e^{j\psi} \frac{p}{p - j\omega} \cdot \frac{1}{pL + R + \frac{1}{pC}} \quad (2)$$

The *time function* $\bar{I}(t)$ may be determined by the generalized expansion theorem [3]. $\bar{I}(t)$ consists of two parts : of the *steady-state component* $\bar{I}_s(t)$ and of the *transient component* $\bar{I}_t(t)$. The steady-state component of the short-circuit current is

$$\bar{I}_s(t) = \frac{\bar{U}}{\bar{Z}} = \frac{U_m e^{j(\omega t + \psi - \varphi)}}{\sqrt{R^2 + (X_L - X_C)^2}} = I_m e^{j(\omega t + \psi - \varphi)} \quad (3)$$

Let us examine the change of amplitude of the steady-state current in case of different compensations. The terms below are to be introduced :

$k = X_C/X_L$ the *degree of compensation*

$h = R/X_L$ the *resistance — inductive reactance ratio* (i. e. R/X ratio)

$$* f(p) = p \int_0^\infty f(t) e^{-pt} dt$$

$I_f = U_m/X_L$ fictive current. (The amplitude of the short-circuit current would equal this value if the inductive reactance alone were taken into consideration.)

On the basis of above, the amplitude of the steady-state short-circuit current may be expressed as follows :

$$I_s = I_m = \frac{1}{\sqrt{h^2 + (1-k)^2}} I_f = c_s I_f \quad (4)$$

where c_s means the correction factor of the steady-state short-circuit current.

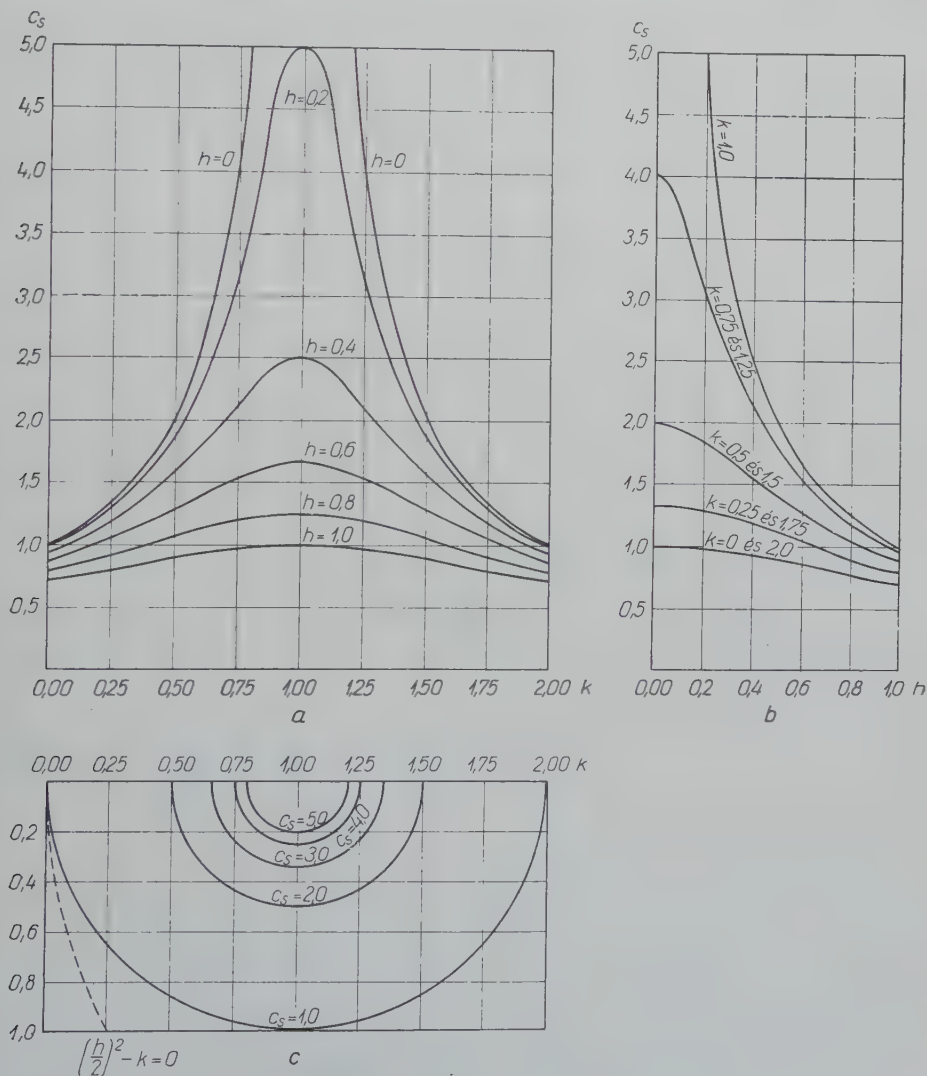


Fig. 2

Figs. 2a, b, c show the coherent values oh h , k and c_s . If the notations

$$\alpha = \frac{R}{2L} = \frac{R}{2X_L} \omega = \frac{h}{2} \omega \quad (5)$$

$$\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = \omega \sqrt{\left(\frac{R}{2X_L}\right) - \frac{X_C}{X_L}} = \omega \sqrt{\left(\frac{h}{2}\right)^2 - k} \quad (6)$$

$$\nu = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \omega \sqrt{\frac{X_C}{X_L} - \left(\frac{R}{2X_L}\right)^2} = \omega \sqrt{k - \left(\frac{h}{2}\right)^2} \quad (7)$$

are introduced we may write the transient component $\bar{I}_t(t)$ in the following form:

a) In the aperiodic case

$$\begin{aligned} \bar{I}_t(t) = & -I_m e^{j(\psi-\varphi)} \frac{1}{2 \sqrt{\left(\frac{h}{2}\right)^2 - k}} \times \\ & \times \left\{ \left[-\left(\frac{h}{2}\right) + \sqrt{\left(\frac{h}{2}\right)^2 - k} + jk \right] e^{(-\alpha + j\nu)t} - \right. \\ & \left. - \left[-\left(\frac{h}{2}\right) - \sqrt{\left(\frac{h}{2}\right)^2 - k} + jk \right] e^{(-\alpha - \beta)t} \right\}. \end{aligned} \quad (8)$$

The short-circuit current transient component of the *noncompensated* system is obtained by the substitutions $k = 0$, $\beta = \alpha$:

$$\bar{I}_t(t) = -I_m e^{j(\psi-\varphi)} e^{-2\alpha t} \quad (9)$$

b) In the aperiodic limit case:

$$\bar{I}_t(t) = -I_m e^{j(\psi-\varphi)} e^{-\alpha t} \left[1 - \left(\frac{h}{2}\right) \omega t \left(1 - j \frac{h}{2} \right) \right] \quad (10)$$

c) Finally, in the periodic case:

$$\begin{aligned} \bar{I}_t(t) = & -I_m e^{j(\psi-\varphi)} \frac{1}{2j \sqrt{k - \left(\frac{h}{2}\right)^2}} \times \\ & \times \left\{ \left[-\left(\frac{h}{2}\right) + j \sqrt{k - \left(\frac{h}{2}\right)^2} + jk \right] e^{(-\alpha + j\nu)t} - \right. \end{aligned} \quad (11)$$

$$-\left[-\left(\frac{h}{2}\right) - j\sqrt{k - \left(\frac{h}{2}\right)^2} + jk\right]e^{(-\alpha - j\nu t)}\Bigg\}.$$

This formula is simplified in case of $R = 0$ and $h = 0$;

$$\bar{I}_t(t) = -I_m e^{j(\psi - \frac{\pi}{2})} \cdot \frac{(1 + \sqrt{k})e^{j\nu t} + (1 - \sqrt{k})e^{-j\nu t}}{2} \quad (12)$$

Here simply $\nu = \omega \sqrt{k}$.

2. Comparison of the short-circuit currents

Now that the time functions are known the short-circuit currents can be compared. Comparison will be effected on the basis of the peak value of the short-circuit current.

It is advisable to keep the complex quantities and to employ a method of graphic design for the determination of peak currents. The steady-state component $\bar{I}_s(t)$ is represented by a vector of constant length rotating with angular velocity ω . The transient component $\bar{I}_t(t)$ may be represented by two vectors of different lengths [see Expressions (8) . . . (12)]. In both cases the initial value of the resultant vector is zero, the short-circuit current starts from the zero value. To determine the peak current for different times, we draft the resultant of the three (in case of $k = 0$, two) vectors and find that of the greatest absolute value.

When drafting, the factor $e^{j(\psi - \varphi)}$ involved in both steady-state component and transient component may be abandoned as a factor not influencing the absolute value of the vectors, but determining only their starting position. On the other hand, the factor I_m figuring in all components, may be taken as a factor of unit length. In this case drafting will not give the greatest peak value, but only a *peak factor* c_t . Knowing the peak factor, we may express the short-circuit peak current as follows:

$$I_c = c_t I_s = c_t I_m. \quad (13)$$

In Fig. 2c the limit curve $\left(\frac{h}{2}\right)^2 - k = 0$ is also shown. It may be read from the figure that the most important cases for practical purposes fall within the territory of $k > \left(\frac{h}{2}\right)^2$. Consequently, for the realization of the design we shall use first of all the periodical solution given by Eq. (11) and, for comparison, the solution regarding the noncompensated case given by Eq. (9).

greatest peak current in the most unfavourable case, i. e., when the greatest short-circuit peak current has indeed developed. Near the maximum value, the resultant diagram deviates but slightly from the arc of a circle. Hence, the peak factor may be determined with sufficient accuracy though the space of time with less precision. Fig. 4 gives the peak factors c_t , in the function of h , with different k parameters.

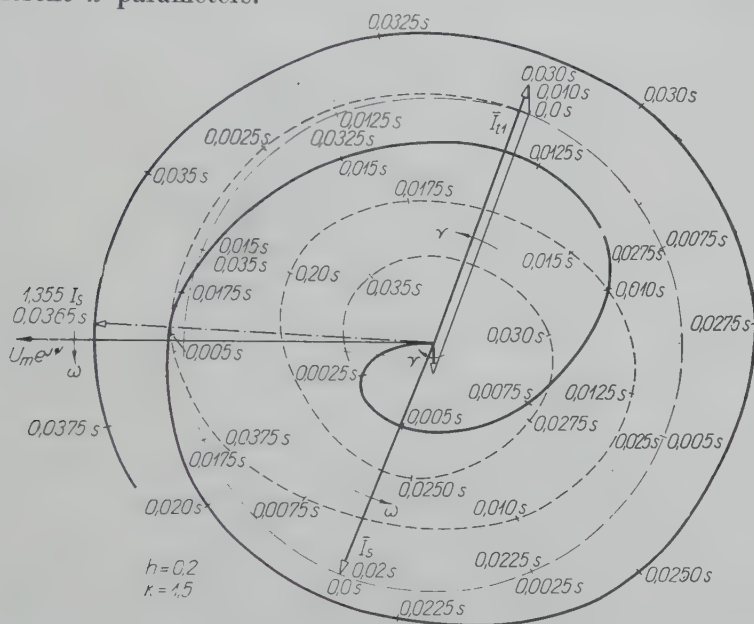


Fig. 3b

Table 1b

Time in sec from the short circuit up to the development of the highest peak current

$k \backslash h$	0,2	0,4	0,6
0,0	0,009	0,00838	0,00791
0,25	0,0195	0,0185	0,0175
0,50	0,028	0,028	0,0275
0,75	0,0565	0,0475	?
1,0	?	?	?
1,5	0,0365	0,0370	0,0375
2,0	0,0175	0,0175	0,0175

Knowing the factors c_s and c_t , the greatest short-circuit peak current may be computed with the formula

$$I_c = c_s c_t I_f = I_f. \quad (14)$$

Fig. 5 shows the *resultant factor* $c = c_s c_t$, in the function of h , with different k parameters.

Making use of factor c , the short-circuit currents may be compared according to different degrees of compensation and different ratios of R/X . To make comparison easier the $\sqrt{2}$ -times, as well as the 2-times value of c_0 in the non-

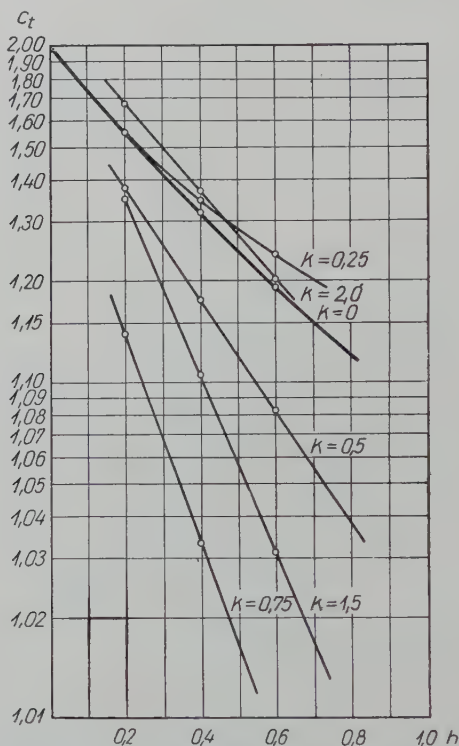


Fig. 4

compensated system (curves $\sqrt{2} c_0$ and $2 c_0$) are plotted on the figure (dotted line). Employment of terms c , c_t and c_s for the comparison of short-circuit currents is shown by an example.

4. Example

Let us compare the short-circuit currents of the system shown in Fig. 6. Calculations were made in relative units referred to a basic voltage of 22 kV and to a basic power of 2 MVA. Results of the calculation are shown in Table 2.

Should a short circuit behind the step-down transformer take place the results obtained show I_s to increase by 2.06 and I_t by 1.68, in case of $X_C = 10.76\%$

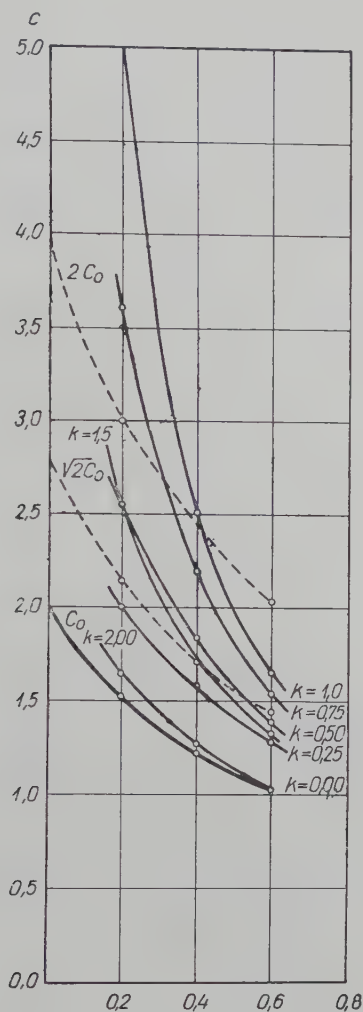


Fig. 5

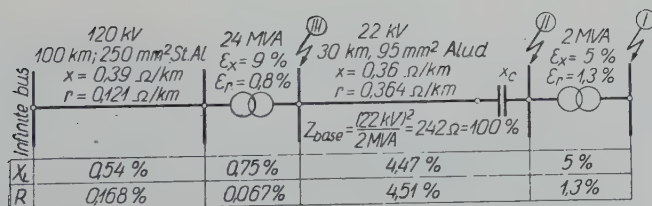


Fig. 6

($k = 1$), while in case of $X_C = 5,38\%$ ($k = 0,5$), the increases of I_s will be 1,52 and that of I_t will be 1,37 as compared to the noncompensated case. If the short circuit takes place at the receiving end of the transmission line, in case of $X_C = 10,76\%$ I_t increases by 1,08, I_i by 1,05, in case of $X_C = 5,38\%$, I_s increases by 1,59 and, I_t by 1,42 as against the noncompensated case. But even these increased currents are considerably smaller than the short-circuit peak current, resp. the steady-state current arising at the sending end of the transmission line.

Table 2.
Calculation of the short-circuit currents

Place of the short circuit	R %	X_L %	X_C %	h	k	c_s	c_t	c	I_f p. u.	I_s p. u.	I_c p. u.
I.	6,045	10,76	10,76	0,562	1,0	1,77	1,0	1,77	9,3	16,46	16,46
I.	6,045	10,76	5,38	0,562	0,5	1,31	1,097	1,433	9,3	12,18	13,32
I.	6,045	10,76	0,00	0,562	0,0	0,86	1,22	1,05	9,3	8,0	9,77
II.	4,745	5,76	10,76	0,824	1,87	0,81	1,09	0,882	17,35	14,05	15,3
II.	4,745	5,76	5,38	0,824	0,935	1,19	1,0	1,19	17,35	20,64	20,64
III.	4,745	5,76	0,00	0,824	0,0	0,75	1,12	0,84	17,35	13,0	14,6
III.	0,235	1,29	—	0,182	0,0	0,98	1,58	1,55	77,5	76,0	120,0

5. Further generalizations

The above calculation started from a three-phase short circuit arising at no-load. The results obtained may be generalized for asymmetrical short circuits too. Thus e. g. in case of a two-phase short circuit the terms c_s and c_t remain unchanged, only the fictive I_t current will increase to its $\sqrt{3}/2$ -times value. Therefore a three-phase short circuit is more unfavourable than a two-phase one.

It is easy to apply the generalization to the single-phase earth-fault (if the resistance R_0 and the inductivity L_0 are considered constant, independently of the frequency), as the positive-, negative- and zero-sequence network must be series-connected. In this case:

$$h = \frac{2R + R_0}{2X + X_0} \quad \text{and} \quad k = \frac{3X_C}{2X + X_0}.$$

As generally

$$\frac{R_0}{X_0} < \frac{R}{X} \quad \text{and} \quad X_0 > X$$

we may count upon smaller values of h and k than in case of a three-phase fault. The current I_f will also be smaller since $2X + X_0 > 3X$. Owing to the fact that with the decrease of h and k the terms c_s , resp. c_t may either increase or decrease, no general statement can be made as to whether a three-phase fault or a single-phase earth-fault is less favourable. In any case conclusions concerning the three-phase fault can invariably be applied to the single-phase earth-

fault, too. The generalization, however, cannot be applied to a two-phase earth-fault, since in this case the negative- and zero-sequence network must be parallelly connected.

We have hitherto examined only such short circuits as may arise at no-load. It remains to find out the character of the influence of the initial load.

The load current at the moment of short-circuit is assumed to be

$$\bar{I}_0 = \frac{U_m e^{j\psi}}{\Sigma Z} \quad (15)$$

where ΣZ comprises the load impedance too. The initial voltage of the capacitor is

$$\bar{V}_0 = -j X_C \bar{I}_0. \quad (16)$$

The Laplace-transform of the short-circuit current in this case is :

$$\bar{I}(p) = U_m e^{j\psi} \frac{p}{p - j\omega} \frac{1}{Z(p)} + \bar{I}_0 \frac{pL}{Z(p)} + \bar{I}_0 j X_C \frac{1}{Z(p)} \quad (17)$$

where $Z(p) = pL + R + \frac{1}{pC}$.

The time function of the first term on the right-side had been determined previously: Eq. (8) ... (22). The time function of the second term for the periodic case :

$$\bar{I}_0 \frac{\left[-\left(\frac{h}{2}\right) + j \right] \sqrt{k - \left(\frac{h}{2}\right)^2} e^{(-\alpha + j\nu)t} - \left[-\left(\frac{h}{2}\right) - j \right] \sqrt{k - \left(\frac{h}{2}\right)^2} e^{(-\alpha - j\nu)t}}{2j \sqrt{k - \left(\frac{h}{2}\right)^2}} \quad (18)$$

The time function of the third term on the right-side is

$$\bar{I}_0 \frac{jk (e^{(-\alpha + j\nu)t} - e^{(-\alpha - j\nu)t})}{2j \sqrt{k - \left(\frac{h}{2}\right)^2}} \quad (19)$$

By this the full solution, considering (3) and (11) is :

$$\begin{aligned} \bar{I}(t) = & I_m e^{j(\psi-\varphi)} e^{j\omega t} - (I_m e^{j(\psi-\varphi)} - \bar{I}_0) \frac{1}{2j \sqrt{k - \left(\frac{h}{2}\right)^2}} \times \\ & \times \left\{ \left[-\left(\frac{h}{2}\right) + j \sqrt{k - \left(\frac{h}{2}\right)^2} + jk \right] e^{(-\alpha + j\nu)t} - \right. \\ & \left. - \left[-\left(\frac{h}{2}\right) - j \sqrt{k - \left(\frac{h}{2}\right)^2} + jk \right] e^{(-\alpha - j\nu)t} \right\}. \end{aligned} \quad (20)$$

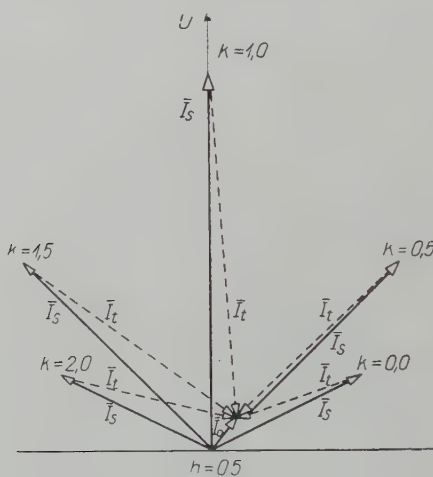


Fig. 7

The steady-state component of the short-circuit current is not influenced by the initial load, but the transient component is changed by it. Fig. 7. shows the vector of the load-current \bar{I}_0 , the steady-state short-circuit current \bar{I}_s and of the resultant transient component \bar{I}_t , for the initial time, with different degrees of compensation k . The transient components are, generally, smaller than in case of faults arising at no-load and exceed this value at very great over-compensation only. As the angle between \bar{I}_s and the resultant vector \bar{I}_t is still close to 180° , the factor c_t under the effect of the initial load generally decreases. In other words, the fault caused at no-load will be more unfavourable.

6. Conclusions

We have studied the increase of the short-circuit current caused by the series capacitor, provided it remains in the fault loop during the whole length of the short circuit. The short-circuit currents of the compensated and non-

compensated systems may be compared by means of the correction factor c_s , the peak factor c_t and the resultant factor $c = c_s c_t$.

1. Conclusions regarding the steady-state short-circuit current may be read from the correction factor c_s (Fig. 2). The more complete the compensation, the more the amplitude of the steady-state short-circuit current at a given ratio of R/X deviates from that of the steady-state short-circuit current in the non-compensated system. At a small degree of compensation, as well as at a great overcompensation the deviation is smaller than at a compensation around 100%. The effect of the ratio R/X must be taken into account too; at a small h value deviation is considerable, at a medium h value it is already smaller and at a very great h value it is insignificant.

2. Conclusions regarding the transient component may be drawn by using factor c_t (Fig. 4). The peak factor c_t to a certain extent changes opposite to the correction factor c_s . At a small degree of compensation and at a very great overcompensation the factor c_t is somewhat greater than the factor c_{t0} of the non-compensated system. Approaching full compensation, the effect of transients decreases, the peak factor c_t comes near to 1. It may also be stated that with an increase of the ratio R/X , the factors c_t themselves considerably decrease, though the deviations are but slightly influenced by the change of h .

3. Conclusions regarding the short-circuit peak current may be drawn by using factor c which includes also the effect of both factors c_s and c_t . The short-circuit peak current of the compensated system may be many times greater than that of the noncompensated system, especially in case of a very small R/X ratio and at a degree of compensation near to 1. Nevertheless, from $R/X = h = 0,4$, even in case of $k = 1,0$, no peak current of more than twice the noncompensated case arises (see Fig. 5). At a small degree of compensation (about $k = 0,25$), the greatest short-circuit peak current in the compensated system is about $\sqrt{2}$ -times that of the noncompensated system, within the range of $R/X = 0,2-0,6$. When $R/X = 0,6$, the peak current will not be greater than $\sqrt{2}$ -times, even at $k = 0,5$.

4. All these comparisons referred to short-circuits arisen at identical points of the compensated and noncompensated system, e. g. at the receiving end. If we compare a short circuit at the receiving end of the compensated transmission line with that at the sending end of the noncompensated system, circumstances become even more favourable in the compensated system from the point of view of both steady-state short-circuit current and short-circuit peak current (see "Example").

5. The above conclusions obtained by assuming a three-phase fault can readily be generalized for cases of a single-phase earth-fault and of a two-phase fault, while the two-phase earth-fault necessitates a separate study.

6. Consideration of the initial load does not affect greatly the above conclusions. Generally, a short circuit caused at no-load is the most unfavourable.

(With the exception of the case of great overcompensation, but practically this is not important).

7. Summing up the above we may state that from the point of view of short-circuit current the series capacitor need not be provided with a protection and by-passed in every case. Although the short-circuit current of the compensated system is always greater than that of the noncompensated system, this increase is not considerable, especially at greater h and smaller k values. For the transformer, the transmission line, the circuit breaker, etc. this increased short-circuit current does not seem very dangerous, as they must endure even the faults arisen before the series capacitor (e. g. the sending end of the transmission line).

7. Overvoltages on the capacitor in series during short circuits

As demonstrated above, the increase of the short-circuit current caused by the capacitor is no great danger to the elements of the system, i. e. the transformer, the transmission line, the circuit breaker, etc. The next question to be examined is, whether the capacitor itself can endure the short-circuit current.

Again the most simple case, the three-phase fault arising at no-load will be studied.

The Laplace-transform of the capacitor voltage may be written in the following form :

$$\bar{V}(p) = \bar{I}(p) \frac{1}{pC} = U_m e^{j\psi} \frac{P}{p - j\omega} \frac{1}{p^2 LC + pRC + 1} \quad (21)$$

Making use of the expansion theorem, the time function of the capacitor voltage e. g. for the periodic case :

$$\begin{aligned} \bar{V}(t) = & U_m e^{j(\psi - \frac{\pi}{2})} \frac{k}{h + j(1-k)} e^{j\omega t} - \\ & - U_m e^{j(\psi - \frac{\pi}{2})} \frac{k}{h + j(1-k)} \frac{1}{2j \sqrt{k - \left(\frac{h}{2}\right)^2}} \times \\ & \times \left\{ \left[-\left(\frac{h}{2}\right) + j \sqrt{k - \left(\frac{h}{2}\right)^2} - j \right] e^{(-\alpha - j\nu)t} - \right. \\ & \left. - \left[-\left(\frac{h}{2}\right) - j \sqrt{k - \left(\frac{h}{2}\right)^2} - j \right] e^{(-\alpha - j\nu)t} \right\} \quad (22) \end{aligned}$$

(By means of the substitutions used for the calculation of the short-circuit current the solution can be written for the other cases, too.)

Neglecting the transient part for the the time being, let us deal with the steady-state component. Its absolute value is :

$$V_s = U_m \frac{k}{\sqrt{h^2 + (1-k)^2}} \quad (23)$$

Let us compare the amplitude V_s of the capacitor voltage established by the steady-state short-circuit current with the amplitude V_0 of the nominal operating voltage of the capacitor,

$$V_0 = I_0 X_C = I_0 X_L k \quad (24)$$

To eliminate the load current I_0 let us write the approaching form of the voltage drop in the noncompensated system :

$$E = \varepsilon U_m \approx I_0 (R \cos \Theta + X_L \sin \Theta) = I_0 X_L (h \cos \Theta + \sin \Theta) \quad (25)$$

where $\cos \Theta$ is the power factor at the sending end of the transmission line.

The division of the above two expressions yields :

$$V_0 = \frac{\varepsilon U_m k}{h \cos \Theta + \sin \Theta} \quad (26)$$

and considering (23)

$$\frac{V_s}{V_0} = \frac{h \cos \Theta + \sin \Theta}{\varepsilon \sqrt{h^2 + (1-k)^2}} \quad (27)$$

Introducing the impedance angle φ_0 of the short-circuit loop for the non-compensated case

$$\cos \varphi_0 = \frac{h}{\sqrt{h^2 + 1}} \quad \text{and} \quad \sin \varphi_0 = \frac{1}{\sqrt{h^2 + 1}}, \quad (28)$$

the quotient of the capacitor voltages may be written as follows :

$$\frac{V_s}{V_0} = \frac{\cos(\Theta - \varphi_0)}{\varepsilon} \cdot \frac{\sqrt{h^2 + 1}}{\sqrt{h^2 + (1-k)^2}} \quad (29)$$

Let us examine the smallest value of the above quotient : The power factor is, generally, between 0,9 and 0,5 corresponding to $\Theta = 26^\circ$ and $\Theta = 60^\circ$, resp. In practice $0,2 < h < 1,0$ and thus $79^\circ > \varphi_0 > 45^\circ$.

Let us consider three cases in detail :

1. $h = 0,2$; $\varphi_0 = 79^\circ$ and $\Theta = 26^\circ$; i. e. $\cos(\Theta - \varphi_0) = \cos 53^\circ \approx 0,6$;
2. $h = 0,6$; $\varphi_0 = 59^\circ$ and $\Theta = 26^\circ$; i. e. $\cos(\Theta - \varphi_0) = \cos 33^\circ = 0,84$;
3. $h = 1,0$; $\varphi_0 = 45^\circ$ and $\Theta = 60^\circ$; i. e. $\cos(\Theta - \varphi_0) = \cos 15^\circ = 0,97$.

For these cases, with different degrees of compensation k , computing the value of

$$\cos(\Theta - \varphi_0) \frac{\sqrt{h^2 + 1}}{\sqrt{h^2 + (1 - k)^2}} \quad (30)$$

it can be stated that except the extreme case of $h = 0,2$, $k = 2,0$, the factor in question is greater than 0,75, moreover, most often greater than 1.

The percentage voltage drop of the noncompensated system is certainly smaller than 30% : $\varepsilon < 0,30$. This means that even in a very favourable case

$$V_s > (2,5 \div 3) V_0$$

The output of the capacitor in case of identical reactance is proportional to the square of the voltage. Consequently, if the series capacitor is not by-passed during fault, it must be oversized at least 6—9 times, which would lead to an uneconomical solution. This unfavourable situation is somewhat improved by the fact that the capacitor may be overloaded for a short time, which, however, leads to an abbreviation of lifetime.

The above proportion would be even worse by taking into consideration the transients. Comparing the transient short-circuit current $\bar{I}_i(t)$ Eq. (11) and the part of the capacitor voltage Eq. (22) in braces, a deviation can be noticed. This seems to require a detailed study of the transient component of the capacitor voltage which, however, could be omitted since the examination of the steady-state component has revealed the necessity of providing the series capacitor with a protective device during short-circuit.

8. Conclusion

We can conclude that the weakest link in the short-circuit loop is the capacitor itself. While the other elements of the network endure even the short-circuit current of the compensated system, the voltage arisen on the capacitor is a multiple of the working value. In most cases it is more economical to design the capacitor for the working voltage and to install a special protective device.

Literature

- (1) Electrical Transmission and Distribution Reference Book. Westinghouse 1950.
- (2) ELSNER H.: Schutz von Seriekkondensatoren gegen externe Störungen und interne Defekte. Bulletin S. E. V. 1952. No. 6. p. 214.
- (3) KOVÁCS, K. P.—RÁCZ, I.: Váltakozóáramú gépek tranziens folyamatai. (Transient processes of a. c. machines.) Akadémiai Kiadó 1954.

Summary

Short-circuit currents arising in power systems compensated by series capacitors and overvoltages of series capacitors are discussed. The aim of the investigation is to answer the question, whether or not the series capacitor in a short-circuit should be by-passed with a protective device in every case.

F. CSÁKI, Budapest, XI., Budafoki út 4—6, Hungary.

ELECTRIC BREAKDOWN AS A PROBABILITY PROCESS

By

J. EISLER

Institute for Electrical Plants and Railways of the Polytechnical University, Budapest

(Received April 5, 1957)

1. Introduction

Recent theoretical and experimental results seem to support the view of those who consider the electric breakdown to be a probability process [1]. The author of this paper made use of this assumption in 1932 in an attempt to establish a theory of the breakdown of liquids [2]. Another theory, much more general, but not very different in basic concepts, was given by Zener [3] in 1934. Experimental evaluation of this theory was carried out lately by McAfee and his co-workers in 1951 [4]. The theories already mentioned use probability concepts in such a way, as is known from the "tunnel effect" of wave-mechanics which serve as a base for them [5].

It is well-known that in this way we generally get theoretical results of statistical character, but in the course of macroscopical measurements we do not find appreciable fluctuations in the measured values. It is, however, also well-known that at the breakdown tests we find a considerable scattering of the measured values, even if a very careful technique is used and the material under test is very homogeneous. This scattering is especially great in the case of impulse voltages, that is when we are obviously faced with the so-called "electric breakdown" which is much less affected by impurities than the breakdown due to thermal instability, the so-called "thermal breakdown".

At first sight, this scattering seems to have nothing to do with the above-mentioned fluctuations, because it is far greater than the latter. Nevertheless, we will see that the existence of some, though indirect, connection between the two phenomena may be assumed.

Obviously, it is worth while to deal with the nature of the scattering, as the dimensioning of insulation is based on the data obtained by breakdown tests carried out on test samples. Since these data show a considerable scattering, it must be decided how to impart the result of the tests to the user, the design engineer. This again requires a thorough discussion of the possible factors responsible for scattering.

We will try to carry out this investigation for the case of the standard breakdown tests. This seems to be justified by two reasons. First, the scattering, being a general phenomenon, must have general causes which are also valid for the standard tests. Secondly, the investigation of the standard breakdown is of high importance for practical purposes.

Consequently, we will investigate the scattering in the case of measurements carried out in accordance with the Recommendations of the IEC. There, the size of the electrodes is specified as follows: the upper electrode is a cylinder of 50 mm diameter and of 50 mm height, its edges are rounded off with a radius of 3 mm. The other electrode is plane, its area is greater than that of the cylinder, the distance between the boundary lines of the two electrodes must be not less than 50 mm. In our tests, described later, the area of the plane electrode was much greater than required by this minimum.

The thickness of the material to be tested should not exceed 1 mm.

2. Experimental results

It is well-known that the electrode arrangement described above yields almost in every case surface discharges before the breakdown of the solid material.

The inception voltage of the glow discharges for plane electrodes is, in air,

$$U_{ig} = \frac{10^{-5}}{(\varepsilon \varepsilon_0)^{0,45}} \cdot a^{0,45} \quad [6],$$

where ε is the relative permittivity of the solid material under test, a is its thickness in cm, $\varepsilon_0 = 0,0885 \cdot 10^{-12}$ Farad/cm.

According to our tests, the formula gives the inception voltage with an accuracy of 2—3% for technically clean surfaces.

If the voltage is raised the brush discharges appear, the inception voltage of which should be

$$U_{ib} = \frac{1,344 \cdot 10^{-4}}{(\varepsilon \varepsilon_0)^{0,44}} a^{0,44} \quad [7],$$

where ε , a , ε_2 are the same as in the previous formula. In our tests, the constant in the numerator varied between $0,8 \cdot 10^{-4}$ and $1,1 \cdot 10^{-4}$, but did not reach the value of $1,344 \cdot 10^{-4}$ given in the formula for U_{ib} . When the surfaces were strongly polluted with dust, then U_{ib} was somewhat diminished and U_{ig} increased. In accordance with the Recommendations, which give no special specification for the condition of the surface the breakdown voltage itself was

but slightly influenced by the condition of the surface. Naturally if, *e. g.*, the surfaces are abnormally wet, this might exert a considerable influence on the breakdown voltage, mainly in case of hygroscopic materials. Measurements were carried out on capacitor paper, oiled paper, two kinds of pressboard and cresol-formaldehyde laminate (bakelite plates) of better and poorer quality. The greatest deviations from the arithmetical mean values were, respectively, 9,5, 7,2, 5,6, 12, 4,3 and 9,7%, calculated from the first 6 measurements. It is worth mentioning that when continuing the tests, in several cases we obtained far greater deviations. This shows that the standardized 6 measurements do not always give a clear picture of the amount of the scattering.

The scattering observed cannot be put down exclusively to the measuring technique, since by measuring the breakdown voltage of air we obtain a scattering in the order of 2% only, although the methods and instruments employed are the same.

Furthermore, variations in the values of the inception voltages U_{ig} and U_{ib} seem to have no immediate influence on the scattering of the breakdown voltages. In case of the capacitor paper and of the oiled paper, the breakdown occurred before the brush discharges appeared.

Tests on glass, under widely varying surface conditions yielded very great differences in the inception voltages, but only a comparatively small change in the mean value of the breakdown voltage and in the scattering of the measured values.

Therefore the reasons of scattering may be supposed to originate partly in the material itself, as also in the case under discussion, that is, when there are surface discharges previous to the breakdown. It may be supposed that for these materials of restricted purity and homogeneity the scattering is partly due to macroscopic impurities.

We shall quote some facts which seem to support our statements.

a) The greatest deviation for pressboard *A*, which is of a better quality and has also a greater electric strength, is some 50% smaller than that for the pressboard *B* of poorer quality. The somewhat defective cresol-formaldehyde laminate showed also a far greater deviation from the mean value than the good one.

b) In case of capacitor paper, the relative scattering at first diminishes with increasing number of layers.

c) It seems to be clear, however, that we get scattering also by breakdown tests on very homogeneous materials, as *e. g.* glass, especially with impulse voltages. The impurities may be *one* of the causes of scattering, but certainly not their *only* cause.

d) We cannot obtain considerably greater mean values of breakdown voltages unless all edge effects are suppressed, that is, no surface discharges occur previous to the breakdown.

Another conclusion that could be drawn from our experiments is that, in accordance with the generally accepted opinion, the mean value of the breakdown voltage is much lower in the presence of surface discharges than without them; though the statistical nature of the phenomenon seems to be the same, and the scattering of the results is identical.

We attempted to investigate some details about the mechanism of the breakdown in the presence of surface discharges, being particularly interested whether or not our previous statement, according to which the breakdown is a probability process, can be applied to this case.

In the meantime an excellent paper was published in "Electrical Energy" about the work of the ERA in the field of insulation research [8], but as it deals with another side of the subject, it will perhaps not be superfluous to describe our results.

It is well-known that in the presence of surface discharges, the breakdown very frequently occurs not between the electrodes, but near the edges or sometimes at a distance from the electrode with the smaller surface. It is also well-known that in this case the breakdown occurs along a surface discharge path, especially at the end of one, because the field is very strong there. It seems therefore that the breakdown is influenced by the shape of these discharge paths.

One might suppose, on the other hand, that the shape of these paths is influenced by macroscopic inhomogeneities on the surface or within the solid material under test. If this were true, the shape and position of these paths should be the same for repeated voltage impulses, or at least nearly the same. The known photographs from surface discharges seem to assert this assumption. It must, however, be considered that these photographs always show a fairly great number of discharges, that have occurred at different times. This is due to the method frequently used to register the discharges on a photographic plate itself. We have carried out a number of experiments with separate impulses at a sequence of about 20 sec, on a bakelite surface, which were photographed separately. The five photos shown in Fig. 1 reveal the more or less different shape of the discharge path under identical circumstances.

One might conclude that the discharge paths do not always choose macroscopic inhomogeneities, because, if so, their shape should remain the same during the repetitions. Quite naturally, we find the same phenomenon with alternating voltages of 50 Hz (Fig. 2), although on these photos we have more discharges that occurred at different times. Fig 2a shows a series of photos taken with a time of exposure of 10^{-2} sec, on Fig. 2b another series is to be seen, taken with a "Zeithupe", in immediate sequence, with a time of exposure of $\sim 10^{-3}$ sec.

Anyway it is obvious that the shape of the paths is different at different times.

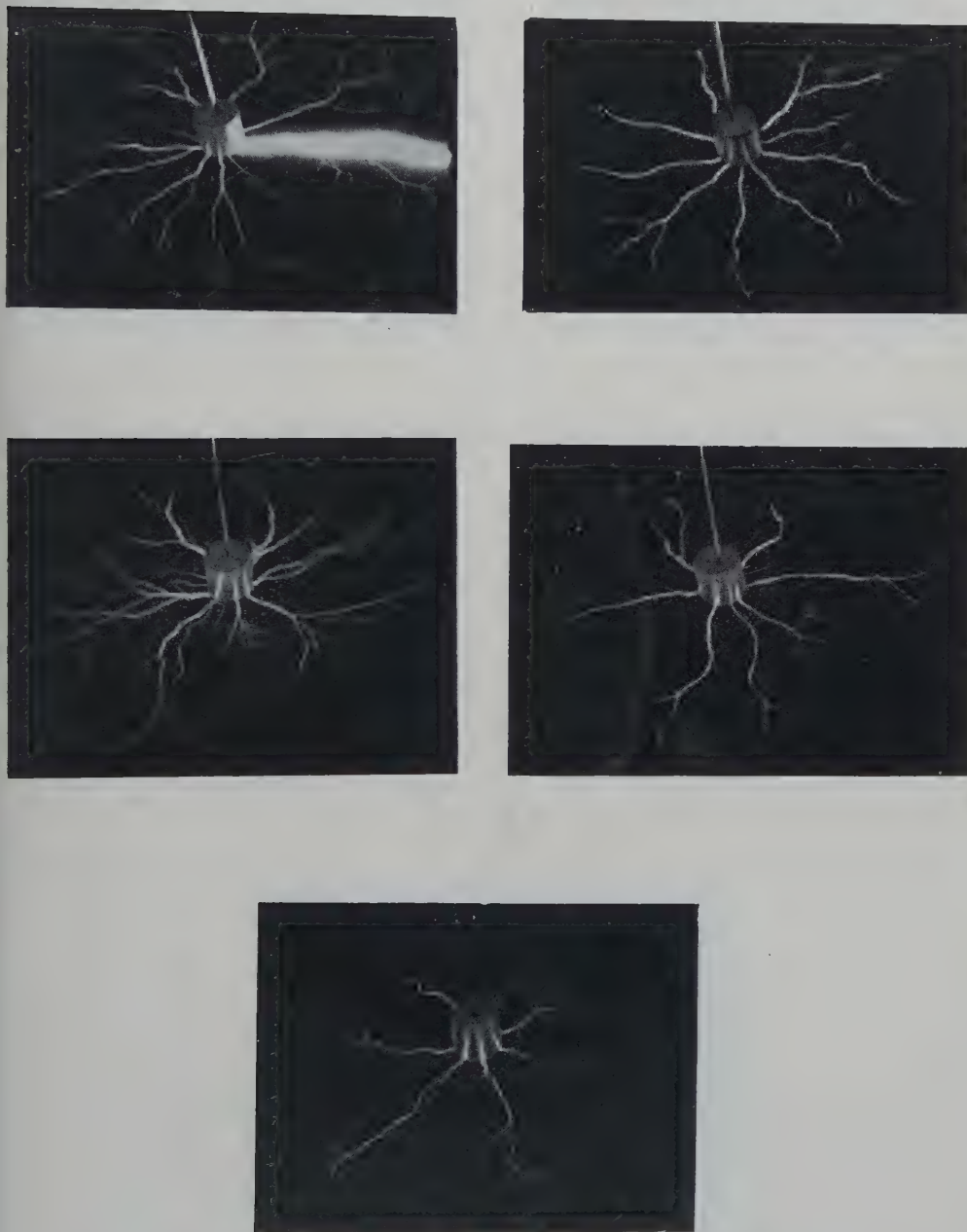
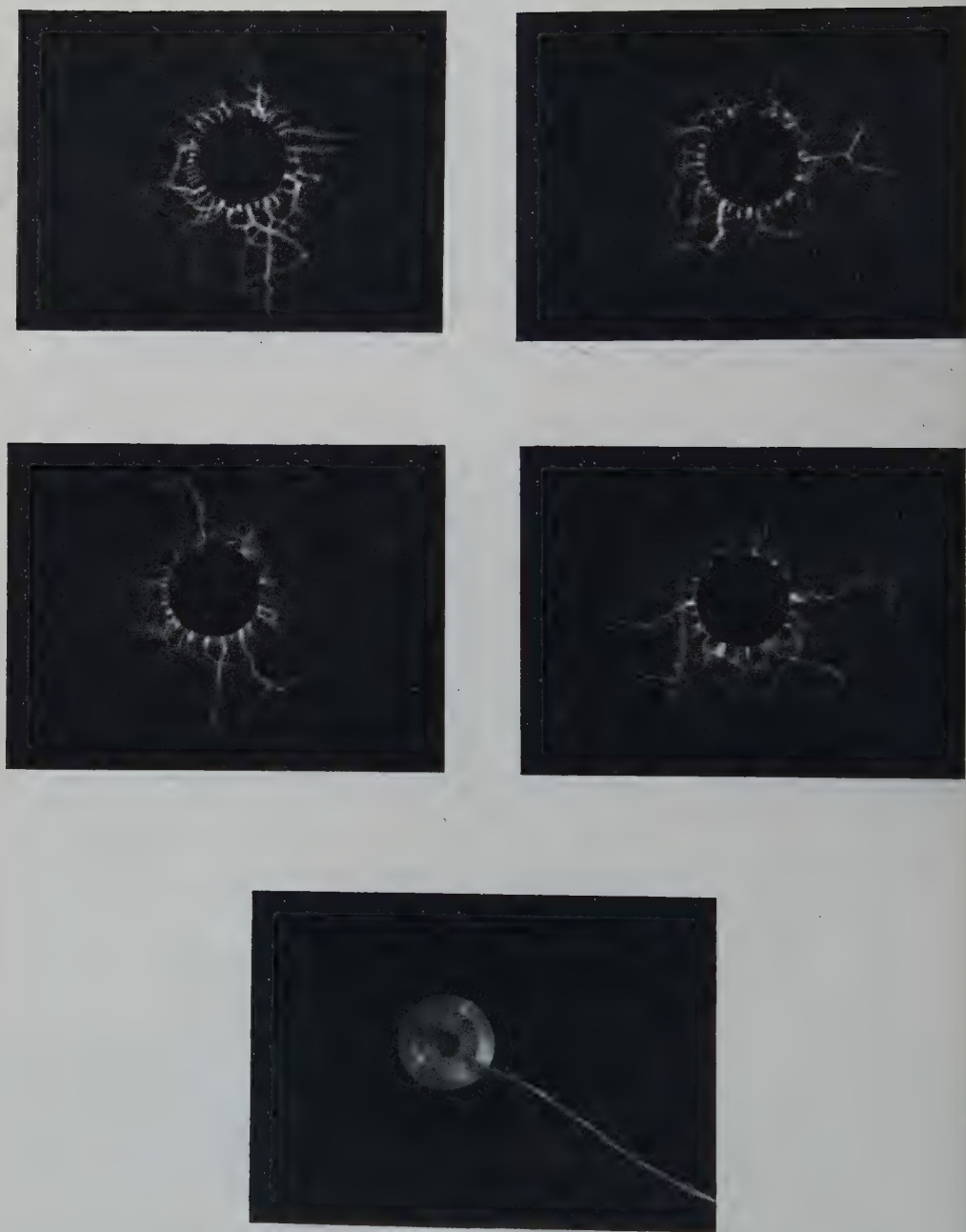


Fig. 1

*Fig. 2/a*

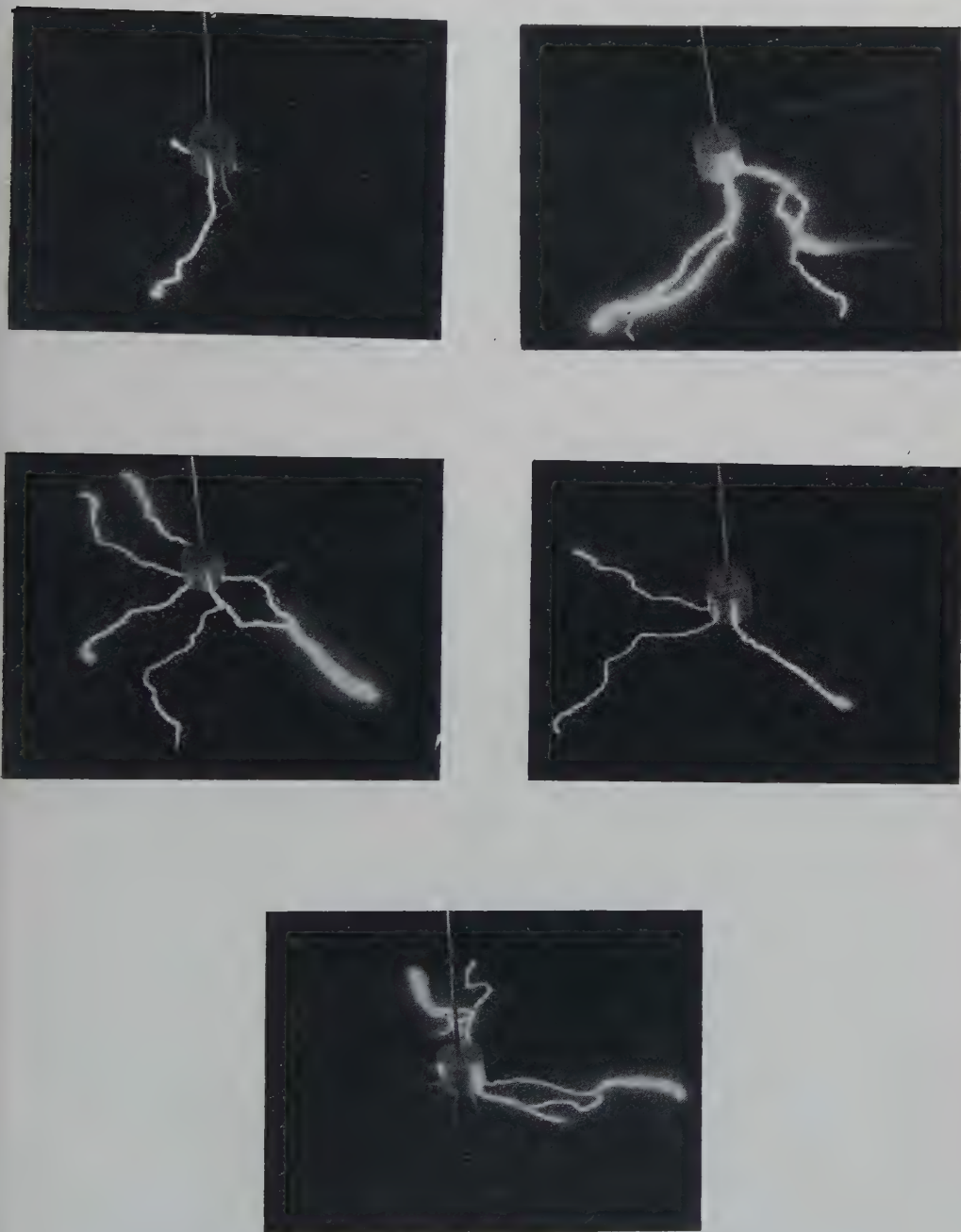


Fig. 2/b

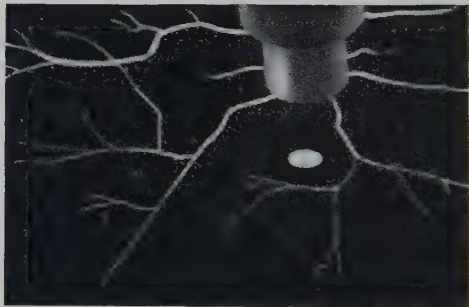
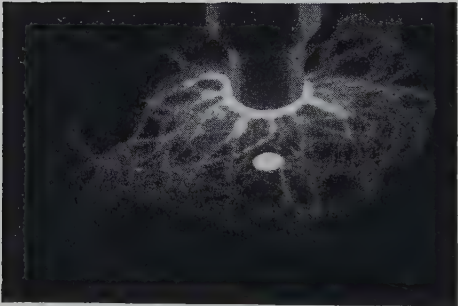
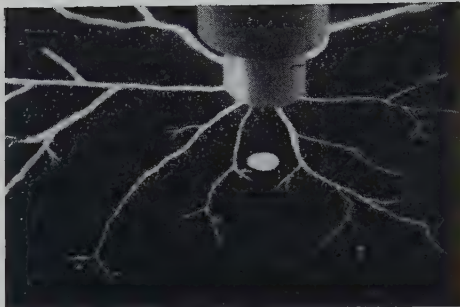
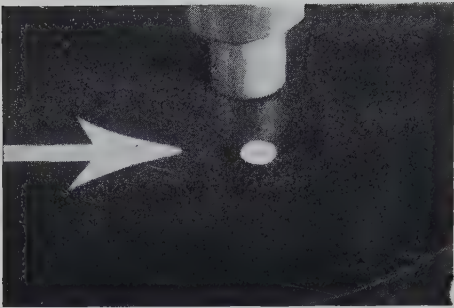


Fig. 3

The maximum distance of the end of these paths from the electrode is, as known, a function of the applied voltage, a fact which is utilized in the impulse voltage measuring instrument called klydonograph.

It can be demonstrated that the discharge paths do not always choose macroscopical inhomogeneities by drilling a hole in the material to a depth of about one third of the thickness. Impulse voltages yield sometimes a discharge path going to the hole, sometimes another going in a different direction (Fig. 3).

These are facts that can be accounted for in different ways. One of the possible explanations may be the following: the instantaneous shape of the

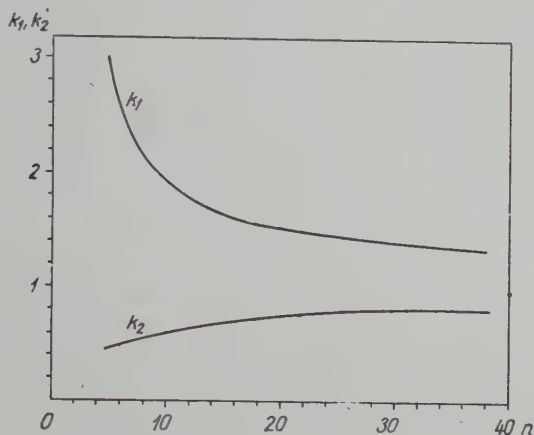


Fig. 4

discharge paths is partly influenced by the fluctuations in the values of the different parameters of the surrounding medium, partly by those of the material under test. If this is so, we can say that the discharge paths are determined by probabilities and, as the breakdown in the material itself is supposed to be a probability process, we may draw the conclusion that the breakdown in presence of surface discharges can also be considered as a probability process. If this is true, we can use the known methods of mathematical statistics for the evaluation of breakdown tests carried out according to the IEC Recommendations.

For the evaluation of our experiments we made use of the method devised by KRONDŁ [9].

The essential of this method is the calculation of the standard deviation

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\bar{U}_b - U_b)^2},$$

where U_b is a measured breakdown voltage, \bar{U}_b the mean value of n measured

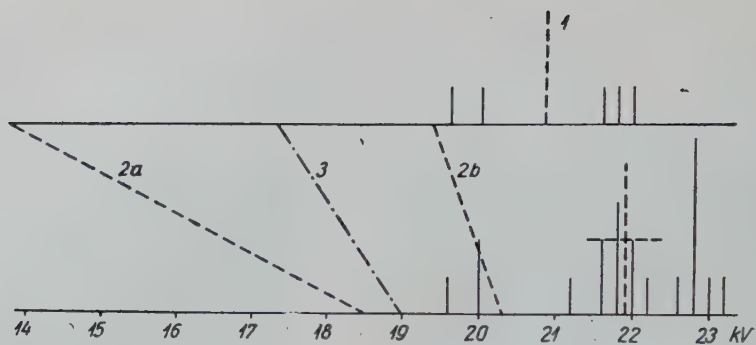


Fig. 5/a

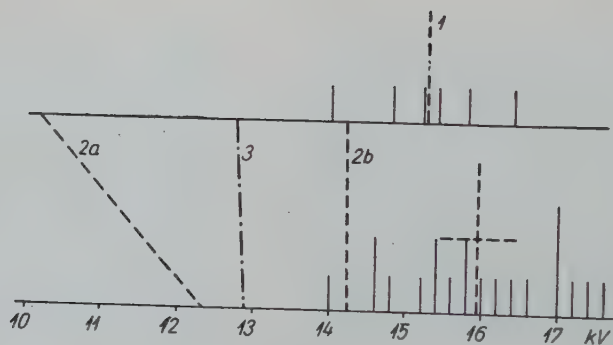


Fig. 5/b

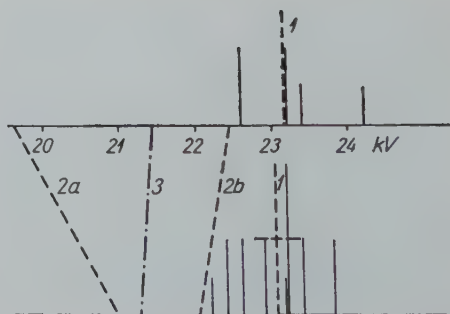


Fig. 5/c

breakdown voltages, that is,

$$\bar{U}_b = \frac{\sum U_b}{n},$$

with these values we get the two limits of the minimum breakdown voltage to be expected as

$$U_b - k_1 2,33 s < \bar{U}_{b\min} < U_b - k_2 2,33 s,$$

where k_1 and k_2 can be taken from graphs as a function of n (Fig. 4). Examples for the evaluation are to be seen in Tables I and II. We have carried out several hundred measurements on the materials mentioned above. The results are to

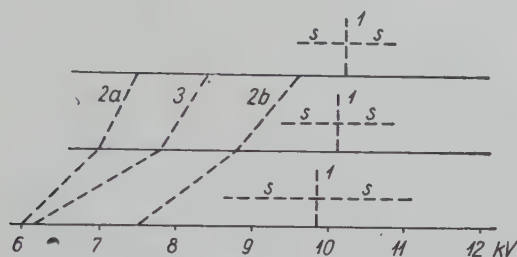


Fig. 5/d

be seen on the diagrams in Fig. 5. Lines Nr. 1 give the mean value, lines Nr. 2a and 2b give the two limits of the minimum breakdown voltage to be expected. Line Nr. 3 shows the values of $U_b - 3s$, which are, as is known, the theoretical minimum values of the breakdown voltage to be expected if we can suppose a regular Gaussian distribution. It occurs sometimes, as seen on the diagram "e" of Fig. 5 that the values of the upper and lower limits of the minimum breakdown voltages are not sufficiently convergent if taken from an increasing number of tests. Thus 20—36 measurements do not seem to yield a reliable lower limit value. It may be useful to make further investigations with this, and perhaps also with other methods, because the evaluation of the probable lower limit of the breakdown voltage from a restricted number of tests is of great practical importance.

3. Conclusions

Theoretical investigations seem to justify the assumption that the so-called "electrical breakdown" is a probability process. The purpose of this paper was to show that also the breakdown in presence of surface discharges, which occurs *e. g.* by using the IEC electrodes, is regarded by author as a probability process.

Table I*
Pressboard A

U_b	$\bar{U}_b - U_b$	$(\bar{U}_b - U_b)^2$	$\bar{U}_b - U_b$	$(\bar{U}_b - U_b)^2$	
20,0 kV	1,9	3,61	0,83	0,69	From measurements 1-6
19,6	1,3	1,69	1,23	1,51	$\sum_1^6 U_b = 125,0$
22,0	0,1	0,01	1,17	1,37	$\bar{U}_b = 20,83$ kV
21,6	0,3	0,09	1,23	1,51	
20,0	1,9	3,61	0,83	0,69	
21,8	0,1	0,01	1,03	1,06	$s = \sqrt{\frac{6,83}{5}} = \sqrt{1,366} = 1,17$ 5,6%
21,8	0,1	0,01		$\Sigma = 6,83$	
22,2	0,3	0,09		$U_{b \min} = 20,83 - \frac{2,6}{0,54} \cdot 1 \cdot 17, 2,33 =$	
22,8	0,9	0,81		$= 20,83 - \frac{7,10}{1,47} = \frac{13,73}{19,36}$ kV	
21,8	0,1	0,01		$\bar{U}_b - 3s = 17,32$ kV	
22,8	0,9	0,81			
23,2	1,3	1,69			From measurements 1-20
21,2	0,7	0,49			
22,8	0,9	0,81			$\sum_1^{20} U_b = 438,0$
22,0	0,1	0,01			$\bar{U}_b = 21,9$ kV
22,6	0,7	0,49			
21,2	0,7	0,49			$s = \sqrt{\frac{17,56}{19}} = \sqrt{0,9242} = 0,961$ 4,4%
23,0	1,1	1,21			
22,8	0,9	0,81			$U_{b \min} = 21,9 - \frac{1,52}{0,72} \cdot 0,961 \cdot 2,33 =$
22,8	0,9	0,81			$= 21,9 - \frac{3,40}{1,61} = \frac{18,50}{20,29}$ kV
		$\Sigma = 17,56$			$\bar{U}_b - 3s = 19,02$ kV

Table II*
Pressboard B

U_b	$\bar{U}_b - U_b$	$(\bar{U}_b - U_b)^2$	$\bar{U}_b - U_b$	$(\bar{U}_b - U_b)^2$	
15,8 kV	0,14	0,02	0,53	0,28	From measurements 1-6
15,4	0,54	0,29	0,13	0,17	$\sum_1^6 U_b = 91,6$
15,2	0,74	0,55	0,07	—	$\bar{U}_b = 15,27$ kV
14,0	1,94	3,76	1,27	1,61	
14,8	1,14	1,30	0,47	0,22	
16,4	0,46	0,21	1,13	1,28	$s = \sqrt{\frac{3,66}{5}} = \sqrt{0,712} = 0,844$ 5,5%
15,6	0,34	0,12		$\Sigma = 3,56$	

* The measurements were carried out by E. NÉMET.

14,6	1,34	1,79
14,6	1,34	1,80
15,4	0,54	0,29
17,4	1,46	2,13
17,0	1,06	1,12
17,0	1,06	1,12
16,6	0,66	0,44
17,2	1,26	1,59
16,2	0,26	0,07
16,0	0,06	—
15,8	0,14	0,02
17,0	1,06	1,12
17,6	1,66	2,75
$\Sigma = 19,49$		

$$U_{b\min} = 15,27 - \frac{2,6}{0,54} 0,844 \cdot 2,33 =$$

$$= 15,27 - \frac{5,11}{1,06} = \frac{10,16}{14,21} \text{ kV}$$

$$\bar{U}_b - 3s = 12,75 \text{ kV}$$

From measurements 1-20

$$\sum_1^{20} U_b = 318,8$$

$$\bar{U}_b = 15,94 \text{ kV}$$

$$s = \sqrt{\frac{19,49}{19}} = \sqrt{1,025} = 1,012 \text{ } 6,4\%$$

$$U_{b\min} = 15,94 - \frac{1,52}{0,72} 1,012 \cdot 2,33 =$$

$$= 15,94 - \frac{3,59}{1,70} = \frac{12,35}{14,24} \text{ kV}$$

$$\bar{U}_b - 3s = 12,91 \text{ kV}$$

Cresol-formaldehyde laminate, good quality, 1 mm thick

23,2 kV	0,13	0,02	—	—
23,2	0,13	0,02	—	—
22,6	0,47	0,22	0,6	0,36
24,2	1,13	1,27	1,0	1,00
23,4	0,33	0,11	0,2	0,04
22,6	0,47	0,22	0,6	0,36
24,0	0,93	0,86	$\Sigma = 1,76$	
22,8	0,27	0,07		
22,8	0,27	0,07		
22,2	0,87	0,76		
22,4	0,67	0,45		
23,2	0,13	0,02		
23,2	0,13	0,02		
23,8	0,73	0,53		
23,2	0,13	0,02		
23,4	0,33	0,11		
23,0	0,07	—		
22,0	1,07	1,14		
23,8	0,73	0,53		
22,4	0,67	0,45		
$\Sigma = 6,89$				

From measurements 1-6

$$\sum_1^6 U_b = 139,2$$

$$\bar{U}_b = 23,2 \text{ kV}$$

$$s = \sqrt{\frac{1,76}{5}} = \sqrt{0,352} = 0,593 \text{ } 2,6\%$$

$$U_{b\min} = 23,2 - \frac{2,6}{0,54} 0,593 \cdot 2,33 =$$

$$= 23,2 - \frac{3,59}{0,75} = \frac{19,61}{22,45} \text{ kV}$$

$$\bar{U}_b - 3s = 21,43 \text{ kV}$$

From measurements 1-20

$$\sum_1^{20} U_b = 461,4$$

$$\bar{U}_b = 23,07 \text{ kV}$$

$$s = \sqrt{\frac{6,89}{19}} = 0,602 \text{ } 2,6\%$$

$$\bar{U}_{b\min} = 23,07 - \frac{1,52}{0,72} 0,602 \cdot 2,33 =$$

$$= 23,07 - \frac{2,13}{1,01} = \frac{20,94}{22,06} \text{ kV}$$

$$\bar{U}_b - 3s = 21,27 \text{ kV}$$

Cresol-formaldehyde laminate, defectuous, 1 mm thick

23,8 kV	0,77	0,59	1,17	1,37	From measurements 1-6	
22,8	0,23	0,05	0,17	0,03	$\sum_1^6 U_b = 135,8$	
22,8	0,23	0,05	0,17	0,03	$\bar{U}_b = 22,63$ kV	
23,4	0,37	0,14	0,77	0,59		
22,6	0,43	0,18	0,03	—		
20,4	2,63	6,92	2,23	4,97	$s = \sqrt{\frac{6,99}{5}} = \sqrt{1,398} = 1,18$ 5,2%	
21,4	1,63	2,65	$\Sigma = 6,99$			
22,4	0,63	0,40				
23,2	0,17	0,03	$U_{b \min} = 22,63 - \frac{2,6}{0,54} \cdot 1,18 \cdot 2,33 =$			
23,4	0,37	0,14	$= 22,63 - \frac{6,61}{1,48} =$		$\frac{16,02}{21,15}$ kV	
22,8	0,23	0,05				
24,4	1,37	1,88	$\bar{U}_b - 3s = 19,09$ kV			
23,4	0,37	0,14				
24,6	1,57	2,46	From measurements 1-20			
22,2	0,83	0,69	$\sum_1^{20} U_b = 460,0$			
23,0	0,03	—	$\bar{U}_b = 23,03$ kV			
23,0	0,03	—				
24,4	1,37	1,88	$s = \sqrt{\frac{18,57}{199}} = \sqrt{0,977} = 0,986$ 4,2%			
23,0	0,03	—				
23,6	0,57	0,32	$U_{b \min} = 23,03 - \frac{1,52}{0,72} \cdot 0,986 \cdot 2,33 =$			
$\Sigma = 18,57$				$= 23,03 - \frac{3,49}{1,65} =$		$\frac{19,54}{21,38}$ kV
				$\bar{U}_b = 3s = 20,08$ kV		

Author omitted a detailed analysis of the mechanism of the breakdown itself in this particular case, since it seems to have been made clear already [8, 10], but tried to show that the breakdown phenomenon might be attributed to fluctuations. The scattering observed seems to be a resultant of these fluctuations.

Author does not consider this to be the only cause of scattering, but it seems very likely that it is *one* of its causes. Should that be true, the use of statistical methods for the evaluation of breakdown measurements, a method for which some examples are given in this paper, is justified.

Literature

1. HAYDEN, I. R.—EDDY, E. W.: *I. A. I. E. E. V.* **41**, p 138 (1922).
2. EISLER, J.: *Zschr. f. Physik* **79**. S. 266 (1932).
3. ZENER, CL.: *Proc. Royal Society* **145**. 523 (1934).
4. MCAFEE and coworkers : *Phys. Review* **83**, 650 (1951).
5. FRENKEL, J.: *Einführung in die Wellenmechanik*. Springer 1929 IV. §. 4.
6. KAPPELER, H.: *Micafil Nachrichten* **5**, 10 (1945).
7. TOEPLER, I.: *Archiv f. El.* **10**. 5, 157 (1921).
8. MASON, J. H.: *Electrical Energy V* **1**. 68—75 (1956).
9. KROND, M.: *Bull. SEV H.* **24** (1953).
10. WHITEHEAD, S.: *Dielectric Breakdown of Solids*, Oxford, Clarendon Press 1951. Chapter IV.

Summary

Author considers the electrical breakdown — also in presence of surface discharges — as a probability process and supposes that at least one cause of the scattering observed is due to this circumstance. If it is true, then the known statistical methods may be used by evaluating the measurements instead of giving the mean value alone. Results of experiments, which seem to justify these assumptions are also given in the paper.

J. EISLER, Budapest, XI., Budafoki út 4—6, Hungary.

EINIGE GESICHTSPUNKTE ZUR BEURTEILUNG VON VERGLEICHENDEN WIRTSCHAFTLICHKEITS- BERECHNUNGEN BEI AUSNUTZUNG VERSCHIEDENER ENERGIETRÄGER

J. KLÁR

(Eingegangen am 1. März 1957)

I

Zu den bisher bekannten Energieträgern ist eine neue, als unerschöpflich erscheinende Energiequelle aufgetreten: die Atomkraft. Die Frage taucht auf, wie sich die Atomkraft auf die übrigen Arten der Energieerzeugung auswirkt, und ob es in Zukunft noch wirtschaftlich sein wird, hauptsächlich auf Kohle (Erdöl, Erdgas) basierte Wärme- und Wasserkraftwerke zu bauen. Offenbar müssen bis zur allgemeinen Verbreitung der Atomkraftgewinnung — wie dies auf der Genfer Internationalen Atomkonferenz festgestellt wurde — noch zahlreiche Fragen geklärt werden. Es ist also mit einer längeren oder kürzeren Übergangsperiode zu rechnen. Nun fragt es sich, auf welche Energieträger bei der inzwischen erwünschten Weiterentwicklung zurückgegriffen, wie also während dieser Übergangszeit die Energiewirtschaft gestaltet werden soll. Berechnungen geben den Nachweis, daß Atomkraftwerke vorläufig sowohl in bezug auf die Investitions- als auch hinsichtlich der Produktionskosten noch hinter den sonstigen Kraftanlagen zurückbleiben. Neben Atomkraftwerken wird man daher einstweilen weiterhin Wärme- und Wasserkraftwerke erstellen. Die Produktionskosten für Wasserkraftanlagen, die die günstigen Naturgegebenheiten ausnutzen, werden voraussichtlich auch in einer fernerer Zukunft mit den Produktionskosten der Atomkraftwerke wettbewerbsfähig sein, um so mehr, da die Atomkraftwerke infolge ihrer Betriebseigenschaften und ihrer im Vergleich zu den Betriebskosten hohen Investitionskosten als Grundlastkraftwerke mit großer Nutzungsdauer gefahren werden müssen. Daher werden Speicherkraftwerke und Pumpspeicherwerke weiterhin zur Übernahme von Belastungsspitzen vorzüglich geeignet sein. Einzelne Länder — die über ausreichende Atomkraftvorräte verfügen — sind gerade heute im Begriff, ihre Wasserkräfte in steigendem Maß auszubauen. Im allgemeinen kann also festgestellt werden, daß bei der Planung der Nutzung einzelner Energieträger die vergleichenden Wirtschaftlichkeitsberechnungen zur Zeit hauptsächlich zwischen den Wärme- und Wasserkraftanlagen durchzuführen sind, die auch heute den überwiegenden Teil der Stromerzeugung zu bewältigen haben.

Die Verfahren zur Durchführung von vergleichenden Wirtschaftlichkeitsberechnungen sind zur Genüge bekannt; doch dürfen nicht nur die in Rechnung gestellten Kosten beachtet werden. Auch die Faktoren, hauptsächlich wirtschaftlicher Natur von verschiedener Art und Bedeutung, sollten sorgfältig erwogen werden, die sich hinter diesen Kenngrößen und Zahlen verbergen. Einzelne Werte der übrigens richtig ausgeführten Berechnungen können ohne dies im Laufe der praktischen Durchführung wesentliche Änderungen erleiden.

Die folgende eingehende Untersuchung setzt sich zum Ziel, die bedeutenderen Faktoren von vorwiegend wirtschaftlichem Charakter festzustellen, die in der Praxis die Realität der Ergebnisse solcher Vergleichsberechnungen gefährden. Diese Tatsachen können am besten klargestellt werden, indem man von der Betrachtung derjenigen Faktoren ausgeht, die sich auf die Wirtschaftlichkeit der Stromerzeugung mittelbar oder unmittelbar fördernd bzw. verringernd auswirken.

Die Wirtschaftlichkeit der Stromerzeugung wird durch folgende Tatsachen *unmittelbar* beeinflusst:

1. vorgesehene Strommenge für eine bestimmte künftige Zeitdauer;
2. wirtschaftlicher Nutzeffekt der veranschlagten Investitionen;
3. die Produktionskosten;
4. wirtschaftliche Schlußfolgerungen des wissenschaftlichen und technischen Fortschritts;
5. bei Wasserkraftanlagen außerdem: anderwertige wasserwirtschaftliche Einrichtungen, die mit der Wasserkraftnutzung zusammenhängen (wie z. B. sogenannte Mehrzweck-Wasserkraftanlagen in Verbindung mit Bewässerungsanlagen oder Schiffschleusen; in einzelnen Fällen mit Hochwasser-Rückhaltebecken kombinierte Wasserkraftwerke).

Mittelbar beeinflussen folgende Faktoren die Wirtschaftlichkeit der Stromerzeugung:

1. allgemeine Planungs-, Bau-, Industrie-, Material- und Arbeitskapazität der betreffenden Wirtschaft;
2. die sogenannte »Sicherheit« der Planung (insonderheit bei Wasserkraftwerken);
3. reale Erfassung der Kohlenvorkommen und Wasserkraftvorräte;
4. Bauzeit (mit besonderem Augenmerk z. B. auf den Bereitschaftsgrad der am Bau teilnehmenden Industrien bzw. auf den gesicherten Import) sowie
5. aus internationalen Abkommen erwachsende Vorteile und Nachteile. (Natürlich können noch einige, weniger wichtige Faktoren in Betracht kommen.)

II

Um eine richtige Entscheidung über die zweckmäßige Nutzung der einzelnen Energieträger fällen zu können, muß zuerst bei einer Perspektivplanung der, während einer vorausbestimmten Zeitdauer erwachsende Strombedarf genau erfaßt werden. Eine Ungenauigkeit in der Berechnung würde das Ausmaß des tatsächlichen Bedarfes fälschen und eventuell zur Heranziehung von Energieträgern führen, die infolge ihrer hohen Kosten bei einem geringeren Energiebedarf ungenutzt bleiben würden. So kann es von der während einer vorausbestimmten Zeitdauer benötigten Energiemenge abhängig sein, bis zu welcher Ausdehnung Wasserkraftwerke mit verhältnismäßig hohen Investitionskosten auszubauen sind.

Zur genauen Bestimmung des künftig benötigten Strombedarfs müssen gesondert berechnet werden: 1. Energiemengen, die mit genügender Genauigkeit vorausszusehen sind (Bedarf, der — dank jahrzehntelangen Erfahrungen ziemlich sicher festgestellt — zumeist nach bekannten Proportionen steigt: beispielsweise Wohnungs- und öffentliche Beleuchtung, Haushaltsstrom, Bedarf der Landwirtschaft und Lebensmittelindustrie usw.) und 2. solche, die nur grob geschätzt werden können. Letztere können erfahrungsgemäß größere periodische Schwankungen aufzeigen — vor allem wenn auch die fernere Zukunft und damit eine mögliche Veränderung der wirtschaftspolitischen Zielsetzungen in Betracht gezogen wird.

III

Besonders eingehende Erwägungen erfordert die Einschätzung des wirtschaftlichen Nutzeffektes bei den in der Stromlieferungsindustrie benötigten Investitionen.

In einer kapitalistischen Wirtschaft, wenn die Stromerzeugung in privaten Händen liegt (ein immer seltener vorkommender Fall), sind für das Ausmaß der in der Stromlieferungsindustrie einzusetzenden Investitionen die Profitmöglichkeiten ausschlaggebend. Der Unternehmer interessiert sich also dafür, wie lange sein Kapital gebunden bleibt, wie es um die Sicherheit der Investitionen steht und nach Erwägung dieser Umstände errechnet er die Höhe der zu fordernden Zinsen. Diese Summe figuriert in den Selbstkostenberechnungen der Unternehmen als Kapitaldienst und unter ähnlichen Titeln. In der Stromlieferungsindustrie ist diese Abgabe verhältnismäßig kleiner als der Ertrag andersartiger Investitionen, dafür gilt er in der Regel als gesichert.

In einer sozialistischen Planwirtschaft oder in einer kapitalistischen Wirtschaft, wo die Stromerzeugung bzw. Stromfernleitung und eventuell die Stromverteilung verstaatlicht sind, unterliegt der wirtschaftliche Nutzeffekt der Investitionen anderen Erwägungen. Da die Aufwendungen auch hier wertvoll sind, ist

bei der Selbstkostenbestimmung in erster Reihe das quantitative Verhältnis festzustellen, aus dem hervorgeht, mit welchem Aufwand an Mitteln ein erwünschtes wirtschaftliches Ziel erreicht werden kann, ferner für welche Zeitdauer diese Mittel gebunden bleiben. (Natürlich können außer diesen auch noch andere Gesichtspunkte zur Geltung kommen.)

Die Entscheidung dieser Frage setzt vor allem eine Prüfung des wirtschaftlichen Nutzeffektes der Investitionen voraus. Eine richtige Einschätzung des Nutzeffektes bei den erforderlichen Investitionen ist für die Zuverlässigkeit der Berechnungen entscheidend, weil es sich nur so beurteilen läßt, mit welchem Aufwand an Mehrinvestitionen eine billigere Erzeugung wirtschaftlich vorteilhaft erzielt werden kann. Mit dieser Frage haben sich vor allem STRUMILIN, HATSCHATUROW, NOKTIN und andere Wirtschaftler eingehend befaßt. Die sowjetischen Forscher haben zur Prüfung des Nutzeffektes von Investitionen im allgemeinen mehrere Anweisungen gegeben, wie z. B. Produktionsvolumen, Rohstoffbasis, Frage der Arbeitskräfte, technisches Niveau der Erzeugung, Ansteigen der Produktivität, Zeitdauer der Realisierung, Rentabilität der Investitionen (Wiedergewinnungszeit), eventuelle Gesichtspunkte des Außenhandels, der Landesverteidigung u. a. m. Die Frage der theoretischen Untersuchungen über den wirtschaftlichen Nutzeffekt einer Investition ist noch lange nicht abgeschlossen. Von den ungarischen Volkswirtschaftlern behandelte K. KÁDAS eingehend die Verfahren der erwähnten sowjetischen Forscher. Unter den bereits angeführten Anweisungen muß die Frage der Wiedergewinnungszeit der Investitionen näher beleuchtet werden. Zwecks Klärung dieser Frage nehme man in Übereinstimmung mit den Betrachtungen von HATSCHATUROW an, daß

$$I_2 - I_1 = I_x \quad \text{und} \quad K_1 - K_2 = K_x,$$

worin I_x die Summe der Mehrinvestition bei einer technischen Entwurfsvariante bedeute und somit $I_2 > I_1$ sei, während K_x die jährliche Einsparung an Selbstkosten infolge der Mehrinvestition I_x ist, demnach $K_2 < K_1$. Die Wiedergewinnungszeit kann also folgendermaßen bestimmt werden: wie hoch ist der Mehrertrag dank der weiteren Senkung der Selbstkosten infolge von Mehrinvestitionen (beim Vergleich von zwei oder mehreren technischen Entwurfsvarianten) oder umgekehrt: welche Summe für Mehrinvestitionen gestattet die erzielbare jährliche Einsparung an Selbstkosten. Der Ingenieur benötigt bei der Beurteilung zweier oder mehrerer technischer Vorentwürfe Anweisungen, die sich auf deren Eigenheiten und Wert beziehen. Jeder Entwurf enthält eine ganze Reihe von Anweisungen der Eigenheiten, wie z. B. in den Fragen der Mangelfstoffe, der Arbeitskraftprobleme usw. Die Angabe der Wertungsangaben dagegen erwarten die Ingenieure von den Volkswirtschaftlern. Nach dem ungarischen Volkswirtschaftler P. ERDŐS können drei Angaben dieser Art empfohlen werden: die spezifischen Investitionskosten sowie die spezifischen Erzeugungs-

kosten; es gibt eine Grenze, über die hinaus zur weiteren Herabsetzung der Selbstkosten die Investitionen bereits erhöht werden müssen, infolgedessen kann zum Zweck der Beurteilung der Wirtschaftlichkeit als dritte Angabe die Wiedergewinnungszeit herangezogen werden. Unter Wiedergewinnungszeit ist die Zeitspanne zu verstehen, nach deren Ablauf die Summe der frei werdenden Mehrmittel aus der Senkung der Selbstkosten den Betrag der durch die Investition gebundenen Mehraufwendungen erreicht.

Bei der Untersuchung des wirtschaftlichen Nutzeffektes von Investitionen müssen auch die möglichen Schwierigkeiten, die aus den Preissystemen erwachsen, berücksichtigt werden. Einzelne Preise der vom Staat gelenkten Preissysteme dürfen in gewissen Fällen nicht ohne Beachtung der notwendigen Berichtigungen angewendet werden (z. B. ohne Abzug der in einzelnen Preisen einberechneten eventuellen Akkumulationen bzw. ohne Hinzurechnung von Dotationen). Ohne diese Korrekturen würden nämlich die Preise des als Grundlage genommenen Materials in vielen Fällen nicht die tatsächlichen Verhältnisse (d. H. die tatsächlichen Gestehungs- bzw. Anschaffungskosten des Materials) wiedergeben.

Die genaue Errechnung dieser Korrekturen stößt nicht selten auf Schwierigkeiten, manchmal ist ihre Bestimmung mit der erforderlichen Genauigkeit überhaupt unmöglich, so daß das Ergebnis der ganzen Nutzeffektuntersuchung unzuverlässig werden kann.

Bei der Preisgestaltung auf dem freien Markt dagegen muß das zu erwartende Ausmaß der stärkeren Preisschwankungen auf dem Markt durch Bestimmung eines entsprechenden Sicherheitsfaktors eingeschätzt werden, wenn die Angaben der Vorentwürfe für eine gewisse Zeitspanne ihre Gültigkeit behalten sollen. In der kapitalistischen Wirtschaft können verschiedene andere bedeutende Änderungen, die als Folge der spontanen Erscheinungen im wirtschaftlichen Gefüge entstehen — wie beispielsweise Wirtschaftskrisen, Veränderungen einzelner Konjunkturzyklen usw. — zu Preisschwankungen führen.

IV

1. Eine weitere wichtige Frage, die auf die Genauigkeit der vergleichenden Wirtschaftlichkeitsberechnungen von Einfluß ist, könnte als »Sicherheit« der *Planung* bezeichnet werden. Sie besteht im wesentlichen darin, daß der Betrag für die benötigten Investitionen (von unerwarteten Ursachen abgesehen) im Zuge der Verwirklichung die geplante Summe nicht übersteige, mag es sich nun um die Erschließung oder Erweiterung einer Kohlengrube oder um die Nutzung einer Wasserkraft usw. handeln. In der Praxis findet man Fehler dieser Art nur allzuoft. Man darf daher weder Zeit noch Geld bei der Durchführung der Entwurfsarbeiten scheuen, denn eine »teure« Planung führt zu billigerer Ausführung.

Besonders bedeutungsvoll ist die Planung, vor allem die Perspektivplanung, für die Entwicklung der Wasserwirtschaft, also auch bei der Aufgabestellung der Wasserkraftnutzung. Diese Aufgaben unterscheiden sich in vielem von den Planungsbedingungen für andere Industriezweige und erstrecken sich im wesentlichen auf folgende Arbeitsgebiete :

a) Aufschließung und Gruppierung der Naturgegebenheiten sowie der gesammelten Angaben, so daß aus ihnen Schlußfolgerungen über den verfügbaren Wasservorrat, die günstigste Verteilung der Wasservorräte gezogen werden können, sowie über Einflüsse, die aus dem Eingriff in den natürlichen Kreislauf des Wassers erwachsen, indem durch den Bau von Werken und Kanalsystemen verschiedenster Art und Anordnung, Änderungen der Wasserstände, in der Wasserverteilung und im Grundwasserspiegel der umliegenden Gebiete vorgenommen werden.

b) Treten die Fragen der Bewässerung, der Wasserkraftnutzung und der Wasserversorgung in den Vordergrund, dann muß die Planung auf die Grundlage der genauen Untersuchung der Durchflußwassermengen gestellt werden. Dementsprechend ist die Veränderlichkeit der Wasserführung von Wasserläufen sowie die Ergiebigkeit der Grundwasserträger genau zu ermitteln. Ein Teil dieser Arbeiten fällt in den Aufgabenkreis der Forschungsanstalten (Pegelbeobachtungen, Durchfluß-, Wasserqualitäts- und sonstige Messungen, Gruppierung der Meßergebnisse, Ausarbeitung der verschiedenen Wahrscheinlichkeitskennwerte mit dem Verfahren der Wahrscheinlichkeitsmathematik, der Korrelationsberechnung usw.). Hierzu gehören ebenfalls die Sammlung und Aufarbeitung meteorologischer Beobachtungen als Ausgangsbedingungen für zahlreiche hydrologische Arbeiten.

Der zweite Teil der Arbeit ist die Aufgabe des Entwurfsbearbeiters. In diesen Bereich fällt die Ausarbeitung von Entwurfsvorlagen zur Nutzung der Wasservorräte eines Landes im Dienste der verschiedenen Zielsetzungen, wie z. B. Berechnungen über die Wasserverteilung, Vorschläge für die Anordnung der dazu erforderlichen Hauptwerke, Kanäle, Staustufen usw. Bei der grundlegenden wasserwirtschaftlichen Planung sollte also der Entwurfsbearbeiter unter Zuhilfenahme der in wissenschaftlichen Instituten in jahre- und sogar jahrzehntelanger Arbeit gesammelten und ausgewerteten Beobachtungen zuerst Anordnungs-skizzen ausarbeiten.

Für die wasserwirtschaftliche Perspektivplanung sind daher sowohl auf dem Gebiet der wissenschaftlichen Forschungen als auch auf jenem der Planung solche wasserwirtschaftliche Aufgaben zu bearbeiten, die vielleicht nur in einer fernen Zukunft zur Ausführung gelangen können. Bei vergleichenden Wirtschaftlichkeitsberechnungen dürfen Investitionskennwerte nur dann als zuverlässig anerkannt werden, wenn diese mit Hilfe der erwähnten Entwurfsarbeit ermittelt worden sind.

2. Eine weitere, streng an die Sicherheit der Planung gebundene Frage bei Wasserkraftanlagen ist, ob wirklich der tatsächliche *Wasserkraftvorrat* in Rechnung gestellt wurde. Die Zuverlässigkeit des nutzbaren Wasserkraftvorrates ist von vielen Faktoren abhängig, wie genaue Ermittlung der Durchflüsse, der Fallhöhe (d. H. genaue hydrologische und geodätische Aufschließung), ferner genaue Bestimmung der Ausbaudurchflußdauer. Diese Faktoren können im allgemeinen mit genügender Genauigkeit bestimmt werden; somit sind auch die den Wirtschaftlichkeitsberechnungen als Grundlage dienenden Leistungen und Energiemengen zuverlässige Werte, und da der Wasserkraftvorrat eine sich auf konkrete Messungen stützende bekannte Größe ist, nicht aber eine »erhoffte« Menge wie der eingeschätzte Kohlenvorrat, ist er dem »verschlossenen Kohlenvorrat« gleichwertig.

3. Für die Genauigkeit der Berechnung ist auch die reale Einschätzung der *Bauzeit* von ausschlaggebender Bedeutung, da diese bekanntlich die Wirtschaftlichkeit der Kraftwerksinvestitionen wesentlich erhöhen oder herabsetzen kann. Deshalb strebt man in der ganzen Welt danach, die Kraftanlagen in der möglichst kürzesten Frist in Betrieb zu setzen. Die Frage der Bauzeitabkürzung hängt mit dem allgemeinen Bereitschaftsgrad der am Bau teilnehmenden Industrien (Material und Arbeitskraft, Entwicklungsgrad der Kraftmaschinenindustrie usw.) aufs engste zusammen. Zur richtigen Bewertung dieser Abhängigkeit seien einige mit dem Kraftmaschinenbau zusammenhängende Fragen näher untersucht, die bereits lange vor Beginn des Kraftwerkbaues geklärt werden müssen. Vor allem besteht in einer sozialistischen Planwirtschaft die volle Möglichkeit, folgende drei Grundsätze rechtzeitig zu klären.

a) Bei welcher Art von Kraftmaschinen (Wärme- bzw. Wasserkraftmaschinen) hat das betreffende Land bereits genügend Erfahrung in der Fabrikation und Handhabung der erzeugten Maschinen?

b) Welche Art von Kraftmaschinen könnten entweder nach Erwerbung ausländischer Dokumentation oder nach Gewährleistung der nötigen Arbeit in Forschungsanstalten oder Fabriklaboratorien mit sicherem Erfolg hergestellt werden, unter Berücksichtigung der Rohstoffvorräte oder Einkaufsmöglichkeiten des betreffenden Landes, wobei auch mit den vorhandenen Facharbeitern, dem Entwurfs- und Konstruktionspersonal gerechnet werden muß.

c) Für welche Art von Kraftmaschinen wäre die einheimische Erzeugung unzumutbar; diese müßten also importiert werden. Die Bedingungen des Imports müßten im vornherein eingehend geprüft und festgestellt werden.

Gegen die richtige Anwendung dieser drei, beinahe selbstverständlichen Grundsätze wird aus verschiedenen Anlässen verstoßen. Die hieraus erwachsenden wirtschaftlichen Nachteile sind in der Regel direkt proportional dem Ausmaß, in dem die als Beispiel angeführten Prinzipien verletzt werden. Infolge der — aus irgendwelchem Grund — nicht zeitgerechten Lieferungen der Kraftmaschinen und der damit verbundenen Verzögerung der Bauzeit verändern sich

nämlich bei der praktischen Verwirklichung sämtliche Angaben der vergleichenden Wirtschaftlichkeitsuntersuchung.

4. Wird schließlich das geplante Wasserkraftwerk auf Grund *internationaler Abkommen* ausgeführt, so werden die darin festgelegten Vereinbarungen — die zumeist ziemlich verwickelte Fragen regeln — die daraus folgenden wirtschaftlichen Vorteile und Belastungen, die Wirtschaftlichkeit des Werkes in bedeutendem Maß beeinflussen. Bei derartigen internationalen Vereinbarungen können besonders in zwei wichtigen Fragekomplexen, und zwar bezüglich der Verteilung der Stromerzeugung zwischen den Unterzeichnenden, sowie hinsichtlich der Verteilung der anfallenden Baukosten für den einen oder anderen Vertragspartner, bedeutende wirtschaftliche Vorteile oder Nachteile entstehen. Diese Frage soll zunächst näher erörtert werden.

Schrifttum

- ERDŐS, P.: A leggazdaságosabb műszaki tervvázlat kiválasztásának néhány kérdéséről (Über einige Fragen der Wahl des wirtschaftlichsten technischen Vorentwurfes). *Közgazdasági Szemle* 2, 203 (1955).
- HATSCHATUROW, T. S.: Grundlagen der Eisenbahnwirtschaftslehre (Kap. III) Moskau 1946.
- HATSCHATUROW, T. S. (ХАЧАТУРОВ, Т. С.): Известия Академии Наук, Отделение экономики и права 4 (1950).
- KÁDAS, K.: A közlekedésfejlesztés gazdasági követelményei (Wirtschaftliche Forderungen der Verkehrsentwicklung). Építőipari és Közlekedési Műszaki Egyetem Tudományos Közleményei, Bd. II, Heft 1.
- KÁDAS, K.: Szovjet módszerek a technikai színvonalat emelő beruházások gazdasági hatékonyságának elbírálására (Sowjetische Verfahren zur Bewertung des wirtschaftlichen Nutzeffektes von Investitionen zwecks Hebung des technischen Niveaus). *Közgazdaságtudományi Szemle* 2—3 (1952).
- KLÁR, J.: Az energiagazdálkodás közgazdasági jelentősége (Volkswirtschaftliche Bedeutung der Energiewirtschaft). Révai Testvérek, Budapest 1946.
- LÉVAY A.: Hőerőművek (Wärmekraftwerke). Budapest 1954.
- LISKA, T.: Kísérleti számítások a beruházások gazdaságosságáról (Experimentelle Berechnungen über die Wirtschaftlichkeit von Investitionen). *Közgazdasági Szemle* 5, 522 (1956).
- MOSONYI, E.: Vízerőhasznosítás (Wasserkraftanlagen) II. Budapest 1953.
- SIEGEL, G.—NISSEL, H.: Nachfrage und Gestehungskosten elektrischer Arbeit. Springer Verlag, Berlin 1938.
- STRUMILIN (Штрумили́н): Известия АН СССР, Отд. экон. и права 3 (1946).
- ТСХРНОМОРДИК, Д. (Чрномордик, Д.): Вопросы экономики 6 (1949).

Zusammenfassung

Die ziffermäßige Genauigkeit der vergleichenden Wirtschaftlichkeitsberechnungen für die Nutzung verschiedener Energieträger hängt in erster Reihe selbstverständlich davon ab, ob die angeführten Positionen sämtlicher (Investitions-, Produktions- usw.) Kosten der Wirklichkeit entsprechen oder nicht.

In vorliegender Abhandlung wurde versucht, auf die — mit den Berechnungsangaben eng in Verbindung stehenden und in ihren Auswirkungen größtenteils meßbaren — wirtschaftlichen Zusammenhänge hinzuweisen, die, wenn sie bei den Berechnungen nicht genügend beachtet worden sind, im Verlauf der praktischen Verwirklichung beträchtliche Abweichungen von den Berechnungsergebnissen zeitigen können. Abweichungen dieser Art können zutage treten, wenn

bei der Perspektivplanung der gesamte Energiebedarf einer bestimmten Zukunftsperiode nicht präzise genug eingeschätzt, der wirtschaftliche Nutzeffekt der Investitionen unrichtig bewertet, wenn bei der Festsetzung der Erzeugungskosten nicht sämtliche kostengestaltende Faktoren mit ihrem tatsächlichen Wert beachtet oder die zu erwartenden kostenvermindernden Einflüsse des technischen Fortschritts nicht real und eingehend erwogen wurden, ferner, wenn die Entwurfsgrundlagen, besonders die Angaben der Perspektivplanung, nicht genau und »sicher« sind, wenn nicht alle Zusammenhänge, die auf die ansonsten richtige Einschätzung der Bauzeit Einfluß haben können berücksichtigt werden, wenn die zu nutzenden Vorräte an Energieträgern ungenau erfaßt oder eingeschätzt und schließlich, wenn die Vor- und Nachteile einer unter eine internationale Regelung fallenden Verpflichtung unrichtig berechnet worden sind.

Eine zuverlässige vergleichende Wirtschaftlichkeitsuntersuchung muß daher einerseits reale zahlenmäßige Tatsachenwerte enthalten, andererseits die Bedeutung aller wichtigen meßbaren wirtschaftlichen Zusammenhänge richtig erfassen und in Rechnung stellen, die die erstere ändern, d. h. erhöhen oder vermindern können.

DR. J. KLÁR, Professor a. D. der Wirtschaftslehre an der Technischen Universität, Budapest, XI. Budafoki út 4—6.

BOOK REVIEW — BUCHBESPRECHUNG

Dr. K. P. Kovács—I. RÁCZ: Transiente Vorgänge bei Wechselstrommaschinen

Akadémiai Kiadó, Budapest 1954, 556 Seiten, 300 Abbildungen

Die zu Beginn unseres Jahrhunderts einsetzende ständige Leistungssteigerung der Synchrongeneratoren und die Kooperation von Kraftwerken war mit einer immer zunehmenden Gefahr der Kurzschlußwirkungen verbunden. Steinmetz war der erste, der diese Erscheinungen im Jahre 1909 in seinem Buch »Transient Electric Phenomena and Oscillations« behandelte.

In Ungarn kannte man aus dem deutschen Schrifttum hauptsächlich die in den zwanziger Jahren erschienenen grundlegenden Arbeiten von RÜDENBERG, BIERMANN und RICHTER. ROSENBERG, PUNGA und GÖRGES beschäftigten sich mit den ständigen Pendelungen parallel geschalteter Synchronmaschinen, wozu der unregelmäßige Gang der damals viel gebrauchten Kolbenmaschinen Anlaß gab. Die Entdeckung des Prinzips der Konstanz der Flüße durch den Amerikaner DOHERTY war für die Erforschung der Kurzschlußerscheinungen grundlegend. Während des Krieges gelangte auch auf diesem Gebiet die führende Position in amerikanische Hände, wo die einzelnen Probleme des Kurzschlusses, der asymmetrischen Belastung und der Stabilität in zahlreichen Artikeln und mehreren zusammenfassenden Werken behandelt wurden.

Diesen vielfältigen Themenkreis bearbeiteten Dr. K. P. Kovács und I. RÁCZ in ihrem erwähnten Buch und sie bringen die unterschiedlichen Auffassungen auf eine gemeinsame Grundlage. Das Buch stellt an das mathematische Wissen des Lesers hohe Ansprüche, es setzt die Kenntnis der Methode der symmetrischen Komponenten voraus und gebraucht sie auch durchwegs, was eine einheitliche Behandlung des Stoffes ermöglicht. Ein kurzer einleitender Abschnitt ist der Laplace-Transformation und der Operatorenrechnung gewidmet.

Von den sodann folgenden beiden Hauptteilen behandelt der eine die Synchron- und

der andere die Asynchronmaschine. Bei den Synchronmaschinen werden nicht nur die Fälle symmetrischer, sondern auch zweipoliger und asymmetrischer Kurzschlüsse besprochen und die Verfasser gehen auch auf die Wirkungen der Dämpferwicklungen und die Berechnung der in verschiedenen Fällen entstehenden Momente ein. Die für den Beweis der Theorie nötigen Meßmethoden werden von Fall zu Fall erläutert. Die mit dem Thema nur in weniger engem Zusammenhang stehenden Fragen der dynamischen Stabilität und der ständigen Pendelungen werden gleichfalls behandelt, und zwar letztere nach einem meines Wissens neuen, aus vorangehendem sich ergebenden Verfahren.

Der zweite Hauptteil erörtert die beim Ein- und Ausschalten bzw. bei anderen Veränderungen der Betriebsverhältnisse auftretenden transienten Vorgänge asynchroner Maschinen, und zwar mit Methoden, die im Schrifttum bisher unbekannt waren.

Im Anhang findet man einige, im Interesse der Übersichtlichkeit in den Hauptteilen weggelassene Ableitungen, Zahlenbeispiele und Kenndaten von Synchronmaschinen.

Das Buch von Dr. K. P. Kovács und I. RÁCZ ist überhaupt das erste, das diesen Themenkreis selbständig aufgefaßt, mit den modernsten Methoden einheitlich behandelt. Die Autoren haben nicht nur die ungarische, sondern auch die internationale technische Literatur mit einem überaus wertvollen Werk bereichert, das auch führende ausländische Fachblätter mit größter Anerkennung besprechen.

Das Werk wurde durch mehrere neue Abschnitte ergänzt ins Deutsche übersetzt und wird im Verlag der Ungarischen Akademie der Wissenschaften demnächst erscheinen.

J. LISKA

Prof. Dr. K. Simonyi: Theoretische Elektrotechnik

VEB. Deutscher Verlag der Wissenschaften, Berlin 1956. Serie Hochschulbücher für Physik, Bd. 20. Übersetzung aus dem Ungarischen. 661 Seiten, 364 Abbildungen, 7 Tabellen.

Das Buch wurde von Prof. Dr. SIMONYI in eine Reihe von fünf Bänden eingegliedert: I. Grundbegriffe—Grundgesetze, II. Makro- und Mikrophysik, III. Praktische Elektrotechnik, IV. Theoretische Elektrotechnik, V. Beispiel- und Aufgabensammlung. Die Bände sind im Ungarischen entweder im Druck oder bereits erschienen.

Die Kenntnisse der Grundbegriffe werden im vorliegenden Band vorausgesetzt, und der Verfasser will hier diese Kenntnisse von einem einheitlichen Standpunkt aus ordnen und zusammenfassen, um dadurch dem Leser den Weg zur Lösung neuer, schwieriger Aufgaben durch Berechnung zu weisen. Nicht die Beschreibungen der physikalischen Erscheinungen oder der technischen Verwirklichungen werden behandelt, sondern die Methoden der Berechnung gestellter Probleme — wie diese zu lösen sind — werden dem Studierenden oder dem Praktiker auseinandergesetzt.

Das Buch behandelt die Elektrizitätslehre deduktiv; ausgehend von den MAXWELLSchen Gleichungen werden einesteiis die Grundgesetze der Elektrizitätslehre abgeleitet, andererseits die theoretischen Berechnungen der verschiedensten Probleme der Elektrotechnik gegeben. Der induktive Weg — der vom Erfassen der experimentellen Tatsachen zur axiomatischen Zusammenfassung sämtlicher Kenntnisse eines ganzen Wissensgebietes führt — wird nur als Einleitung benutzt.

Der erste Teil enthält einen Überblick jener Begriffe, durch welche die Erscheinungen der Elektrodynamik beschrieben werden. Es ist dies sowohl eine Rekapitulation wie eine Zusammenfassung des allgemeinen Verhaltens von Größen, wie Feldstärke, Induktion usw. Im weiteren enthält dieser Teil eine Wiederholung der experimentellen Faktoren und theoretischen Überlegungen, die induktiv zu den MAXWELLSchen Gleichungen führen.

Die mathematischen Grundbegriffe der Vektoranalyse, Vektoroperationen und ihre Umkehrung sind ebenfalls erläutert, deren Kenntnis bei der Lösung der Maxwell'schen Gleichungen unumgänglich sind. Hier werden die Bedingungen zusammengefaßt, die zur eindeutigen Beschreibung von Vektorfeldern nötig sind. Spezielle Funktionen oder Rechenmethoden, die dem Studenten weniger geläufig sind, werden meist an Ort und Stelle ihrer Anwendung besprochen. So finden wir eine Erläuterung der Operatorenbildung in allgemeinen orthogonalen Koordinaten, der BESSEL'schen Funktionen, der LEGENDRE'schen Funktionen, der konformen Abbildung, der ellip-

tischen Funktionen, der LAPLACE-Transformation usw.

Der zweite Teil — die Elektrostatik — ist hauptsächlich der Lösung der LAPLACESchen Gleichung gewidmet. Das Buch zeigt die Möglichkeiten, die der Lösung schwierigerer, von technischer Praxis jedoch öfter gestellten Aufgaben zur Verfügung stehen. Es werden die Methoden der Lösung dieser Randwertaufgaben in der Ebene — hier wird vor allem die Vielfältigkeit der Anwendung der konformen Abbildung gezeigt — und in zylindersymmetrischen und kugelsymmetrischen elektrostatischen Feldern erörtert.

Im dritten Teil sind einige voneinander mehr oder weniger unabhängige Teile zusammengefaßt. Einesteiis wird die Berechnung des magnetischen Feldes stationärer Ströme, das Verhalten von quasistationären Stromkreisen besprochen.

Sehr ausführlich wird der Skineffekt behandelt. Lösungen werden angeführt, die in technischer Hinsicht wichtig sind, wie der zylindrische Leiter, der geschichtete Leiter, die Stromverdrängung in den Nuten der elektrischen Maschinen mit Diagrammen über den Verlauf des Widerstandes und der Selbstinduktivität. Der Abschnitt über Fernleitungen gibt eine Zusammenfassung der speziellen Begriffe und die Berechnung dieser Größen.

Die Behandlung von Einschaltvorgängen wird in einem gesonderten Abschnitt behandelt und die Anwendung der LAPLACE-Transformation wird ausführlich gezeigt.

Im vierten Teil werden Erscheinungen der Elektrodynamik behandelt, bei denen der Verschiebungsstrom $\partial D/\partial t$ einen nicht zu vernachlässigenden Einfluß auf das Zustandekommen des Magnetfeldes hat. Nach Hervorhebung des Unterschiedes, der zwischen der Fortführung elektromagnetischer Wellen in solchen Medien besteht, deren Leitfähigkeit Null oder endlich ist, folgen ausführliche Beschreibungen der ebenen Wellen, Zylinderwellen und Kugelwellen, ohne dabei nach der Art und Weise der Erregung dieser Wellen zu fragen. Die verschiedenen Anordnungen zur Erregung der Wellen werden gesondert behandelt, wie die linearen Antennen und Antennensysteme, weiterhin der GOUBAUSche Oberflächenleiter, Kugelantenne, Doppelkonusantenne, wobei auf die vorhergehenden Lösungen von E und H nur zurückgegriffen wird.

Mehr in sich abgeschlossene Teile sind die Kapitel über Hohlraumstrahlung, d. h. Hohlleitungen und Hohlraumresonatoren. In diesen Abschnitten ist überall der Verlauf von

$\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$, $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ und der abgestrahlten Leistung abgeleitet, bei Antennen meist noch der Strahlungswiderstand, bei Hohlleitungen der Wellenwiderstand und die Eigenwellenlänge, bei Resonatoren der Gütefaktor.

Der letzte Abschnitt befaßt sich mit der Ausstrahlung von Flächenelementen, Streuung, Beugung im elektromagnetischen Feld, sowie dem BABINETschen Prinzip und weist auch auf die Problemstellung und die Möglichkeit ihrer Ausnutzung hin.

Beim Durchsehen dieser Kapitel finden wir überall einen Anschluß an die Fragen der

modernen angewandten Elektrotechnik. (Einschaltvorgänge, Hohlleiter, Resonatoren, Flächenstrahlung usw., um nur einige Beispiele zu erwähnen.) Sie sind den seit langer Zeit angewandten Problemen beigelegt. (Elektrostatische Probleme, HELMHOLTZ-Spule, Dipol-Strahlung usw.) Diese Vielseitigkeit bereichert sehr das in seiner Behandlungsweise so einheitliche Buch. Die Anschaulichkeit der Gedankengänge sowie die ausdrucksvollen Abbildungen machen auch die schwersten Abschnitte dem Leser leicht verständlich. Den Wert des Buches erhöht noch die schöne Ausstattung, für die der Verlag Sorge trug.

P. KOSTKA

SOME QUESTIONS OF INDUCTION-TYPE METERS

By

B. E. F. KARSA

Chair for Electrical Machinery and Measurement

Up-to-date electrical measuring instruments have to fulfil the following requirements

1. safety
2. high accuracy (low relative errors)
3. low costs of manufacturing
4. stability
5. quickness of operation
6. low internal consumption
7. insensibility against variations of internal parameters of the measurement and against overload
8. insensibility against variations of external parameters
9. beauty

From these the fifth may be neglected in the case of the meters; and the third contradicts the rest, so a reasonable compromise is inevitable.

According to the above requirements, meters should be judged by their metering-equation. As it is generally known the driving torque $M_{i,u}$ of an induction-type meter is proportional to the electric power ($P = IU \cos \varphi$) flowing through the meter into the consumer's electric circle; the braking torque M_b generated by the permanent magnet works in opposite direction and is proportional to the meter's Ω angular velocity and to the square of the braking flux Φ_b .

That is why the electric power flowing through it is measured by the angular velocity of the meter, whereas the electric energy flown through it during the same time by its returns made in a time-interval:

$$\int_{t_1}^{t_2} P dt = k \int_{t_1}^{t_2} \Omega dt \text{ and } W_2 - W_1 = (v_2 - v_1) K$$

All circumstances that may cause any variation in the values of the above-mentioned torques and those exciting further torques change the measuring equation of the meter, i. e. its constant which had been found correct.

The said measuring equation may be deduced in different ways without, however, ignoring some simplifying assumptions. It does not seem to be correct to use analogies of a rotating magnetic field (there is no magnetic flux rotating around the axis of the meter's disc) so in the case of a meter we had better discard such expressions as "number of poles" or "synchronous angular velocity".

The driving elements of a meter are the current coil and magnet and the voltage coil and magnet, on the one hand, and the rotating disc, on the other. These magnets are independently excited but their fluxes develop in each other's immediate neighbourhood, in narrow intimacy. Each of them excited alone develops a flux passing through the disc with lines, in general, perpendicular to the disc, varying with the frequency of the net inducing eddy currents of the same frequency (Figs. 1 and 2).

The orbit and strength of the eddy currents are subject to continuous changes and so is the resultant, penetrating the disc, of the flux derived from the two fluxes excited in the two magnet-cores.

The resulting magnetic flux together with the eddy currents generates a force which acts in the plane of the disc (though may from time to time act even outside of it). With varying direction and value this force

makes a fan-like (oscillatory) movement in the disc:

$$F = f(t)$$

is a periodical function of time, its frequency being the double of that of the supplying voltage. Because of the inertia of the disc, the average resultant of the force should be calculated for one period of the voltage. If it is zero or if it passes through the axis of the disc, the medium driving torque is

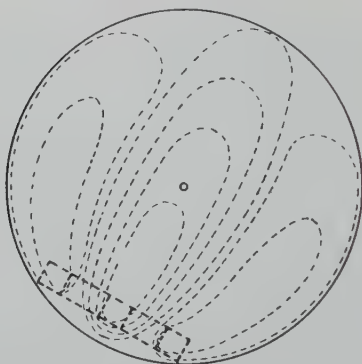


Fig. 1. Eddy currents induced by the voltage coil alone

zero, in all other cases there exists a — positiv or negativ — driving torque. Even “shaking forces” leaving the plane of the disc are likely to act.

An alternating magnetic flux generates zero medium-torque with the eddy currents induced if the average orbits of the eddy currents are symmetrical to the straight line connecting the average centre of the flux with the centre of the disc. By disturbing this symmetry a driving torque can be generated; this is the task of the cooperation of current coil and voltage coil (the simplest example: compensation of friction by the aid of a small asymmetry in the air gap of the voltage magnet).

In the first years of development, the safety of continuous operation was the only requirement that could be fulfilled. The first

specimens (Fig. 3a, 3b) had an error limit of $\pm 5\%$ in case the intensity of the current was between 10% and 120% of the basic current and the power factor $\cos \varphi$ not less than 0.5. Their weight was surprisingly great (12 to 14 kgs). Bláthy, of course, was aware of the principles governing the operation of the meter, but when manufacturing was started there was no possibility of taking into account all the parameters of the measurement.

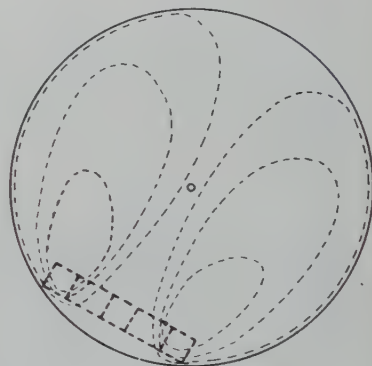


Fig. 2. Eddy currents induced by the current coil alone

The first attempt was aimed at decreasing the error-limits; having recognised the influence of friction Bláthy wished to decrease weight and angular velocity of the rotating system. Soon he solved the first problem but as to the angular velocity he could not find the necessary braking magnet, so there was a delay in the solution. A further step was the compensation of friction (by the mentioned small asymmetry) and together with it the reliable elimination of rotation at no load (braking filament).

The power of the driving eddy currents has a braking effect. This recognition led to further development: special forming (magnetically shunting) applied in the core of the current coil compensates the braking effect of the eddy currents induced by the flux of the current coil.

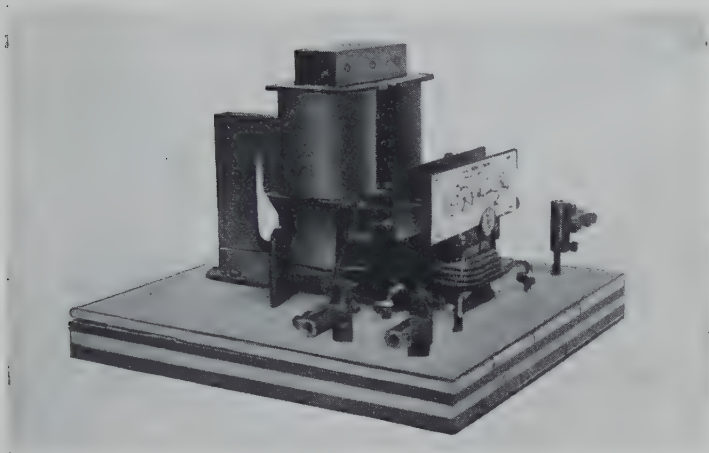


Fig. 3a. Bláthy's first induction-type meter, left: braking magnet, centre: voltage coil, right: current coil

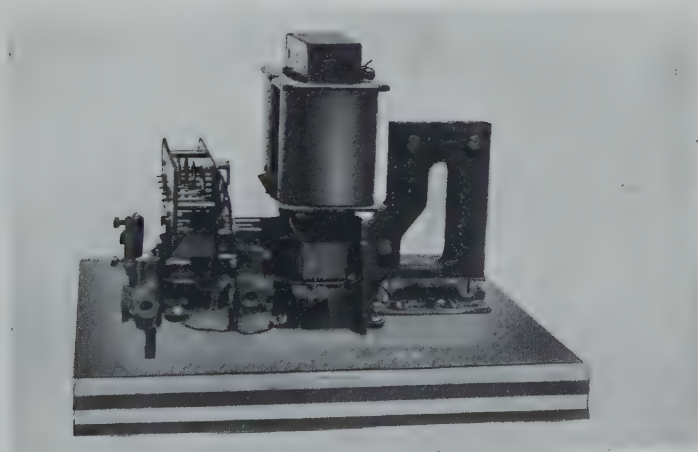


Fig. 3b. Bláthy's first induction-type meter, left: current coil, centre: voltage coil, right: braking magnet

Attention was soon turned to the internal and external parameters of the measurement: variations of voltage frequency and ambient temperature influence in different ways the working of the meter. Clear knowledge of the principles governing the operation of the meter has been gained thanks to the diligent work of many investigators — with prominent

and the medium value with rms. voltage and current

$$M = K IU \sin \psi$$

where ψ is the phase-angle of the two fluxes. If by the aid of construction this angle and the phase angle φ of the load can be made comple-

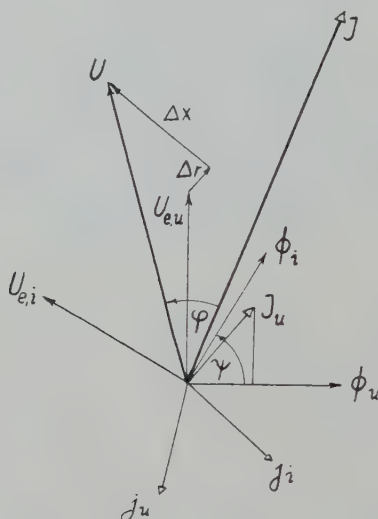


Fig. 4. Vector-diagramm of an induction-type meter

cooperation of Rogowsky and his collaborators. Based on the vector-diagramm of the meter (Fig. 4) they deduced the equation of its driving torque, calculating with component fluxes (flux Φ_u and Φ_i of voltage coil and current coil resp.) and with component eddy currents (j_u and j_i). So the instantaneous value of the tangential force being

$$F_{i,u} = c_i \Phi_i j_u + c_u \Phi_u j_i = c_i \Phi_i c'_u f \Phi_u + c_u \Phi_u c'_i f \Phi_i,$$

Rogowsky has shown that, in the above equation,

$$c'_i = c_i \quad \text{and} \quad c'_u = c_u$$

and therefore the instantaneous value of the torque is

$$M_{i,u} = c_1 \Phi_i \Phi_u f = c_2 i_u$$

mentary, i. e., $\varphi + \psi = 90^\circ$, the medium torque is

$$M = K IU \cos \varphi$$

This deduction is shorter than to observe the eddy currents and the force connected with them, but, taking into account tangential forces only, fails to remind us that the forces may leave the disc, periodically causing undesirable "shaking" effects.

The influence of the variation of voltage, frequency and power factor and the changes in the temperature can be read from the vector-diagramm of the meter.

The influence of the variation of voltage and frequency is clearly shown in the transformer equation: the induced voltage U_e is

$$U_e = 4 k f f N B_m A$$

where only $U_e f$ and B_m (voltage, frequency and maximum induction) are variable; the

first two being parameters of the load's circle, their variation causes a variation of the flux density B . This, in turn, changes strength and phase angle of the exciting current I_u ; this again, alters the flux Φ_u in the voltage coil and its phase angle to the load's current I . The error so caused is expected to be considerable if the power factor $\cos \varphi$ is small.

Thus the influence of the changes in temperature can be traced. The resistance of the disc might vary nearly without any further result since it alters driving and braking torque — even the braking losses of the driving eddy currents at the same ratio. Altering of temperature and resistance of the voltage coil will have a considerable effect upon the internal phase angle of the meter (upon the one between voltage flux and current flux). Alteration of the internal temperature of the meter involves changes in the parameters of ferromagnetic materials too (in first line the flux of the brake magnet which will decrease with rising temperature and so will the braking torque, by the square). The latter change might be compensated by applying a magnetic shunt of proper qualities at the poles of the brake magnet.

Such requirements as durability and stability of the meter have set new and difficult tasks to construction and manufacture. Competition in reducing costs caused further difficulties. The problem became a pure technological one. All artifices of mass production were applied, new and better materials used in searching for best solution.

The distribution of electrical energy all over the whole earth has brought forth two further requirements: transportability of the meters and their resistance to aggressive climates. Correct working in overloads is the third, raised by the average consumer.

Transportability determines the mechanical properties of the meter: resistance against acceleration, shaking and shocks within reasonable limits without any deformation, tear and wear or breaking of the moving parts. The solution is a mere technological one by utilizing the best construction materials (pivots, gears, shaft etc). The dynamically correct construction of the moving parts is

of no less importance (possibly small masses and inertiae); and at last the properly solid and stiff internal holding construction and a strong and stiff case are equally important.

Resistance to aggressive climate and other harmful effects (dust, insects, fungi, bacteria etc.) may be also obtained by technological procedures. General interest is focused now on this question but experiments and experience are insufficient as yet to form definite opinion as to the methods to be applied in construction and manufacturing. The use of extraordinarily resistant basic materials, protecting and covering materials, excellent fillings is inevitable. It is also obvious that a meter cannot preserve its correct measuring abilities unless its insulation, its mechanical and electromagnetical properties remain unaltered.

A meter has to bear overloads thermically and mechanically, without its measuring abilities decreasing under a given (narrow) limit. Overload can be caused by increasing voltage and current. Only a small rise of voltage is permitted (e. g. 10 per cent above nominal). In everyday practice overload is a current increase above the nominal (basic) value; so we speak of meters "built for fourfold load" (for $I = 4 I_b$).

The thermal effect of an overload ascertains itself first in the temperature rise of the overloaded coils. Its influence upon the voltage coil has partly been discussed. However, the rise of voltage is connected with an increase of the field density and of the exciting current and so with a further change of the internal phase angle of the meter. — The alteration of the resistance of the current coil has no consequence in the phase position of the (forced-on) current, though owing to the increase of field-density, the magnetical shunting in the core of the current coil (at high overloads) might change in an undesirable direction. This circumstance may be corrected e. g. if the magnetic shunt of the current-core is made of two different iron-materials (one saturated at lower densities and the other at high ones only).

The overloadability of the meter can be increased by diminishing the local tempera-

ture-rise of the coils, provided the thermal transductivity of the case does not simultaneously change. This can be achieved e. g. by doubling the driving system (by a diametral action of a couple of forces, without

friction. At present this leads to an expensive solution because of the friction of the gearing but it will obviously be practicable as soon as the mass-production of precision gears at moderate costs will be solved.

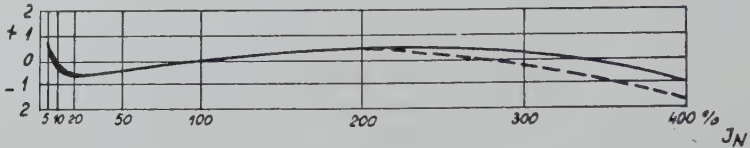


Fig. 5a. Diagramm of the relativ errors of an overloadable meter

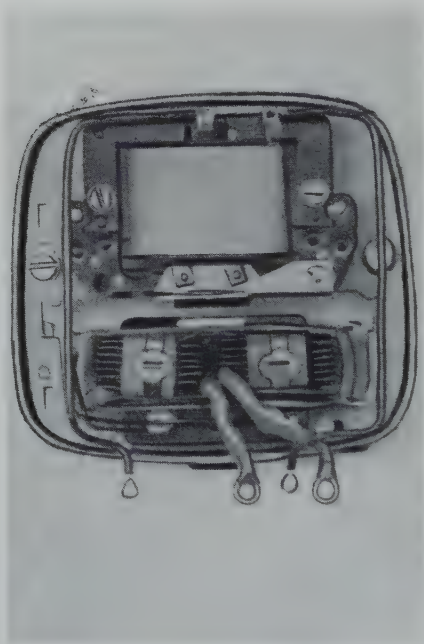


Fig. 5b. Up-to-date overloadable meter

an increase of the torque) increasing thereby the cooling surfaces.

The simultaneous decrease of the internal dimensions may also be taken into consideration (i. e. decreasing the driving forces without diminishing the braking ones) if, we succeed in proportionally decreasing

This solution must ensure the curve of relative errors to remain within the limits recommended by the I. E. C. as shown in the curve of errors (Fig. 5a) of a meter made in Hungary. The considerations exposed above may lead to more simple constructions at moderate costs.

SOME RESEARCH PROBLEMS OF »GANZ« WATER TURBINES*

By

Cs. FÁY

Research Engineer of the Hungarian Academy of Sciences at the Department for Hydraulic Machines of the University of Technical Sciences, Budapest

The industrialization increasing all over the world at a pace never seen before demanded a very significant increase of the energy supply. Hence the requirements raised against hydraulic machines including water turbines are steadily growing and, at the same time, calling for higher quality. The 90 years old manufacture of Hungarian water turbines (the firm *Ganz*) has also been faced with growing tasks the solution of which has in many cases been made possible only through experimental research. Such a multitude and variety of questions arose that they could not be solved by the *Ganz* Works themselves, and, therefore, the scientific ability and the laboratories of the University of Technical Sciences, Budapest were resorted to. A cooperation in scientific research has been achieved especially with the Department for Hydraulic Machines and with the Department for Fluid Mechanics.

In this brief discussion we shall deal in the first place with the adjustable-blade turbines manufactured to meet the demands in Hungary.

In case of the adjustable-blade turbines a machine unit may be divided into three distinct parts, such as the scroll case, the guide vanes and runner, and the draft tube. It is generally known that all of these parts involve such problems as can be solved — besides theoretical calculations and considerations — by means of experiments only.

I. The water flows to the turbine runner through the scroll case and the guide vanes. The water inlet is of confusor-type therefore the head loss in it is low. If, however, the velocity of the flow from the scroll case to the guide vanes is not uniform and is changing also its direction, the direction of the relative flow to the individual blades of the runner is varying in time: it becomes periodically pulsating. This phenomenon not only results in a reduction of the efficiency of the runner but may lead also to unpleasant operating disturbances: the unequally loaded runner may wear the lower guide bearing to oval, may ruin the stuffing box etc. Therefore it is of major importance to give a good shape to the scroll case.

The scroll case (Fig. 1) designed on the basis of theoretical considerations (for a constant vortex) was tested with air and water at the institute for Hydraulic Machinery. The effect of adjusting the guide walls having adjustable end, built into the entrance flume was checked on different scroll cases.

After suitable adjustment of the guide walls the investigations were continued on the equipment for turbine experiments (a larger scale model) of the *Ganz* Works. This time the adjustment of the fixed guide vanes was also carried out. Based on the data of previous measurements the angle and spacing of the fixed guide vanes were determined and checked by velocity distribution measurement.

* A considerable contribution to this paper was made by Mr. Ernest Trenka, Chief of the Department of Hydraulic Machines at the *Ganz* Works who was prevented from completing the manuscript by his trip abroad

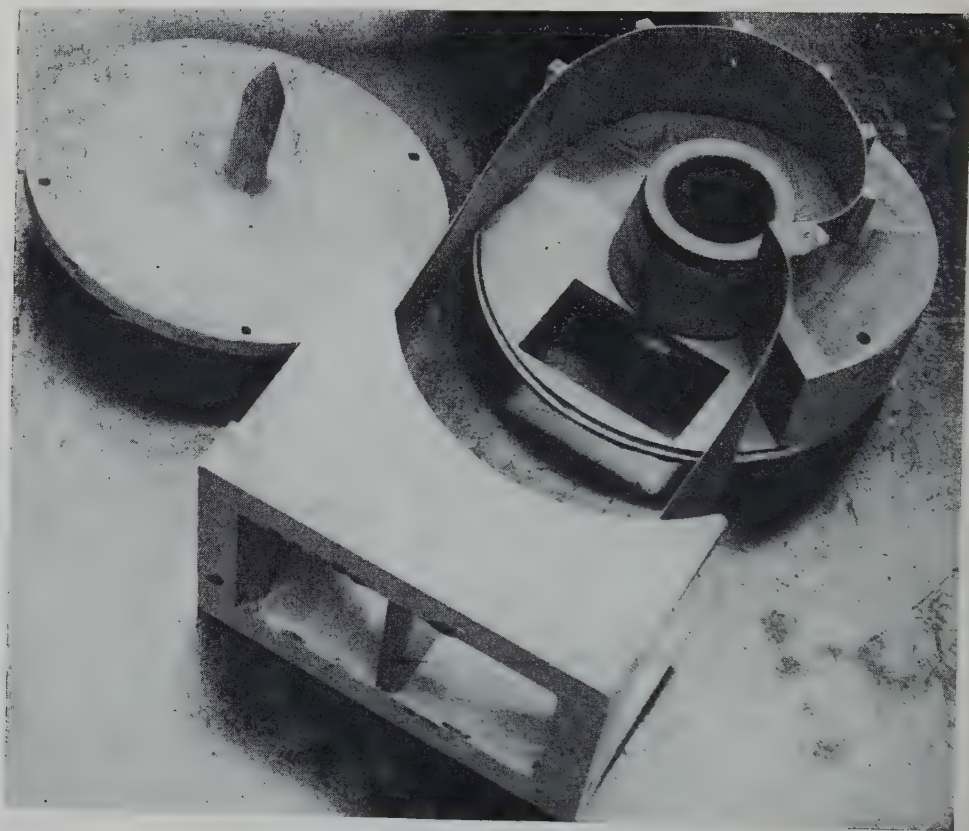


Fig. 1. a) Adjustable scroll case for investigations with air

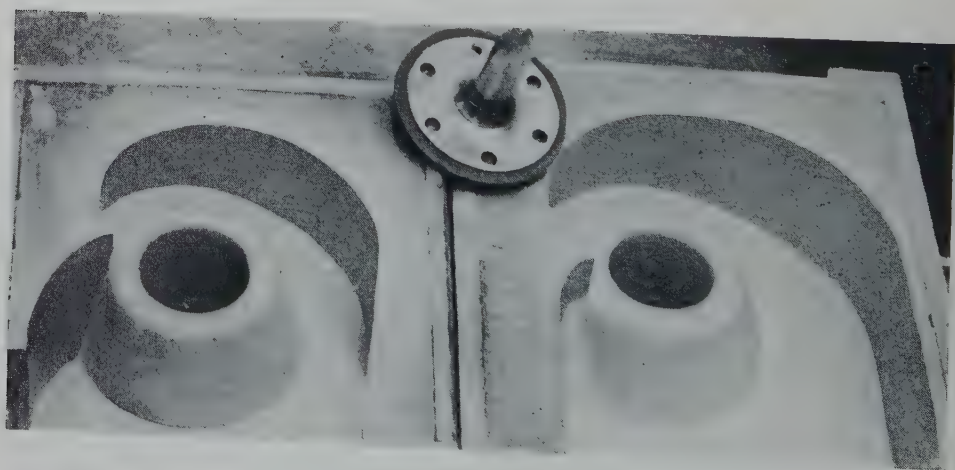


Fig. 1. b) Scroll cases for investigations with water

After this measurement small corrections were made on the vane angle adjustment whereby the optimum adjustment could be found. In Fig. 2 the distribution of meridian and vortex velocities before and after adjustment is shown plotted on the axis circle of the guide vanes. This measurement was carried out without guide vanes.

with a view of determining the lattice-characteristics [3]. Tests of assembled runners were made with the following equipments built in the turbine experiment laboratory of the Ganz Works: an equipment built between open water levels for measuring efficiency and a station built in a closed system for determining the cavitation characteristics.

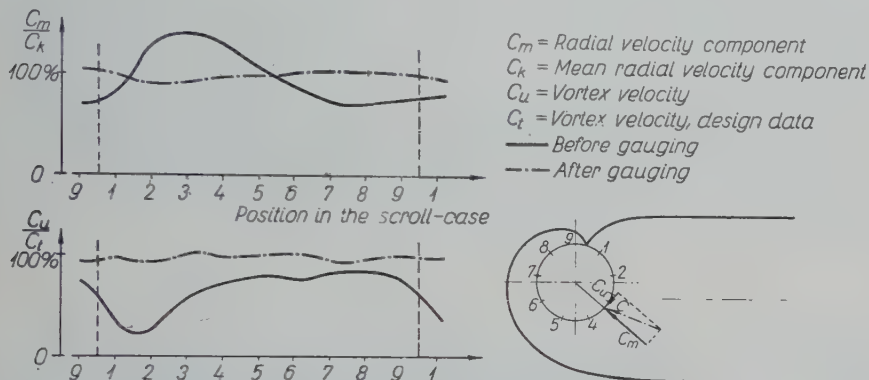


Fig. 2. Velocity distribution at the place of the guide vanes

II. The design of the runner is of extraordinary significance for both efficiency and cavitation characteristics of the turbine. The basis of designing the runner is the blade element (airfoil).

The Institute for Fluid Mechanics developed under the leadership of Professor GRUBER new types of airfoils which can bring about a specified pressure distribution thus being particularly advantageous in view of avoiding cavitation. [1]

For measuring the cavitation characteristics of blade elements an up-to-date cavitation-tunnel was built at the Institute for Hydraulic Machinery under the leadership of the late Á. G. PATTANTYUS [2]. By means of this tunnel the results of theoretical calculations are checked.

The hydrofoils in the turbine runner are arranged in lattice. In addition to the theoretical calculation of the lattice-correction, experimental measurements were carried out with air at the Institute for Fluid Mechanics

III. The kinetic energy of the flow leaving the runner will be reconverted into potential energy in the draft tube. In case of high specific speed turbines from 20 to 40 per cent of the total energy enters the draft tube, therefore the head loss of the draft tube has to be reduced to a minimum. For determining the correct shape of the draft tube extensive series of experiments were carried out.

a) Conditions were investigated with air at normal inflow.

b) A vortex was generated with a small turbine (Fig. 3) and the efficiency was determined at different operating conditions with ten different types of draft tube.

c) By building the best types into the testing station of the Ganz Works accurate measurements were made on a larger-scale model assembled with different runners [4].

It is to be mentioned that an attempt was made to substitute the runner with a stationary lattice for testing the draft tube. The data obtained in this way could not, however, be

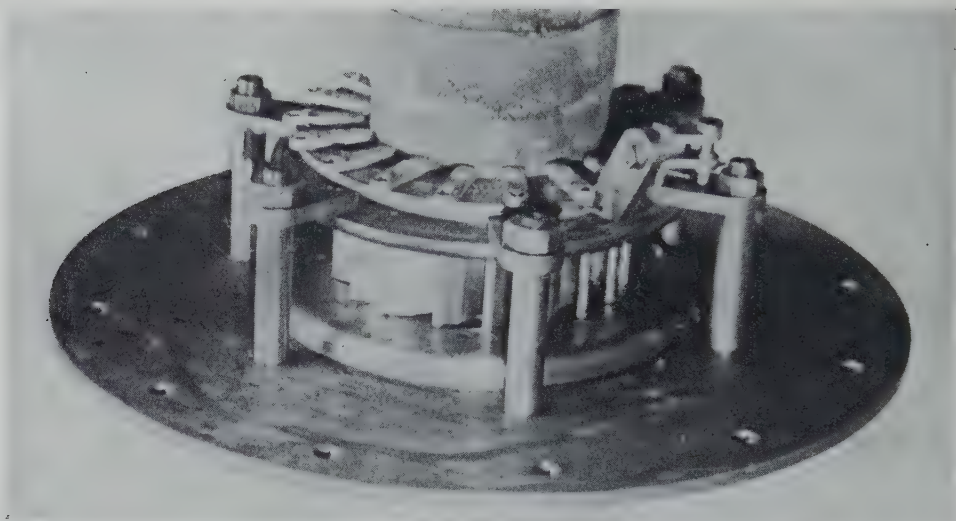


Fig. 3. Small-scale turbine for generating vortex a) guide vanes



Fig. 3. b) runner with turbine cover

compared to the actual conditions, therefore this method was dropped.

IV. Based on the measurement data of the model the expected characteristics of the prototype turbine can be predetermined. For calculating the efficiency of the prototype turbine a great number of formulae are given in the literature. Detailed investigations were made also in this field mainly based on the loss analysing measurements of Kvyatkovsky [5].

In this formula the value of A is still a function of the parameters n_{11} and Q_{11} . By denoting the parameters of the maximum efficiency point Q_{11}^* and n_{11}^* and introducing the variables $x = Q_{11}^*/Q_{11}$ and $y = n_{11}/n_{11}^*$ the curves shown in Fig. 4 are obtained. In the figure the approximations of Ackeret ($A = 0,5$), Hutton-Blackstone, Voith, Escher Wyss and the curve determined by us based on the measurements of Kvyatkovsky are also indicated. We

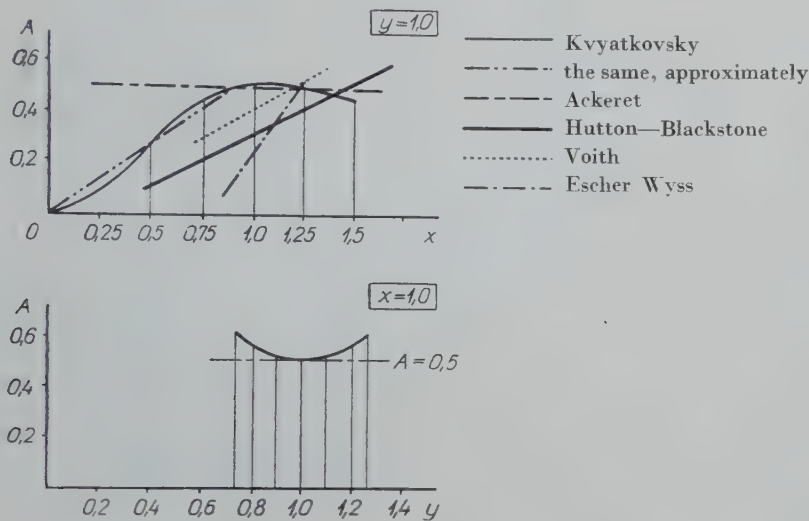


Fig. 4. Comparison of formulae for calculating efficiency

As the final result of the investigation a formula was obtained which gives results falling within the range limited by the data given in the literature not only for the maximum efficiency point of the complete characteristic including isoefficiency curves but also for a sufficiently wide section of the curve. If, in the course of calculating the efficiency, the mechanical losses are eliminated and the volumetric efficiency is assumed to be constant for both the model and the prototype turbine, then the following formula is obtained for the rate $v = \delta/\delta_{\text{mod}}$ of the head losses ($\delta = 1-\zeta$):

$$v = A + (1-A) \cdot (\text{Re}/\text{Re}_{\text{mod}})^{1/5} \cong A + (1-A) \cdot (D/D_{\text{mod}})^{1/5}$$

approximated Kvyatkovsky's curve by the following formula:

$$A = 0,555 x [1 + 3,2 (y - 1)^2] \text{ for } x < 0,9$$

$$A = 0,5 [1 + 3,2 (y - 1)^2] \text{ for } x > 0,9$$

The range of validity is $0,75 < y < 1,25$. The formula holds true only for adjustable-blade turbines.

A special importance is given today to this question by the fact that it is expedient to carry out the acceptance measurements of turbines of very great discharge also on the basis of model measurements. Therefore, if in the course of model measurements, detailed loss-analysing measurements (mechanical, volumetric, scroll case, guide vanes, runner losses, furthermore friction, outlet and boundary

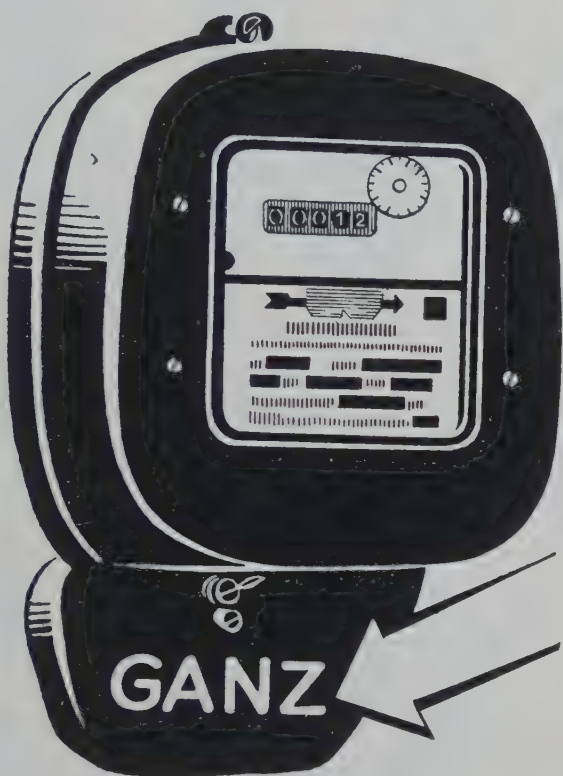
layer losses of the draft tube) are made and the losses can reliably be separated, an absolutely reliable and accurate efficiency calculation can be carried out, the error of which can even be reduced to fall within two limits and is smaller than the ± 2 per cent error usual at the acceptance measurement of the prototype turbine. This method is on the one hand less expensive than the acceptance measurement, on the other hand it also gives information as regards flow conditions in the turbine.

V. Midget water-power plants are of considerable significance in this country. In the design of these plants low-cost and simplicity are decisive factors. The Ganz-Mignon turbines were designed especially for this purpose. The development of this type was carried out through no model measurements but the assembled prototype of the actual machine itself was tested. It was an interesting task to work out experimentally the optimum design — in view of control and efficiency — of the adjustable head cover used in place of the adjustable guide vanes and the adjustment to maximum sensitivity of the direct-acting mechanical speed regulator. The result is: the midget plant operating alone at

constant head gives practically constant voltage at the cable end connection.

REFERENCES

- DR. GRUBER: Blade Section Design in Bladed Hydraulic Machines. Publ. of Dept. VI of Acad. Sc. Hung., Budapest, VI, 1, 1952.
 — DR. GRUBER: Blade Section Design in Axial-Flow Hydraulic Machines. Acta Technica Acad. Sc. Hung. Budapest, V, 1952. — DR. GRUBER: On the Problem of Judging Blade Sections of Axial Hydraulic Machines. Acta Technica Acad. Sc. Hung., Budapest, VII, 1953.
 GEREY: The Present State of Experimental Research into Cavitation. Publ. no. 5 of Hydr. Mach. Dept. of Univ. of Techn. Sc., Budapest, 1949. Cs. Fáy: Cavitation-tunnel of the Univ. of Techn. Sc., Budapest. Manuscript for the Publ. of the Dept. VI of Acad. Sc. Hung.
 DR. GRUBER: Approximate Determination of the Velocity Field of Incomplete Vortex Lines. Publ. of the Dept. VI of Acad. Sc. Hung., Budapest, XVIII, 1956.
 CS. FÁY—E. TRENKA: A selection from the subject-matter "Methods and Instruments of Hydraulic Machine Research". Manuscript no. 3315, Mérnöki továbbképző, Budapest, 1955.
 КВУАТКОВСКИЙ: Рабочий процесс оцевой гидротурбины. Машгиз. 1951.



METER FACTORY GANZ

EXPORT DEPARTMENT

BUDAPEST V., Guszev u. 25.

P. O. B. 53. Budapest 172.

Phone Number 113-252, 129-416.

Telex Number 698.



Instruments for laboratories and research institutes

Measuring instruments for the textile industry

Instruments for power-plant control

Medical and surgical instruments

Medical and surgical apparatus

Electric measuring instruments

Medical and dental furniture

Electro-medical apparatus

Veterinary instruments

Electronic instruments

Geodetic instruments

Dental supplies

A m p o u l e s

Exporters :

METRIMPEX

Hungarian Trading Company
for Instruments

Letters : Budapest 62.

P. O. B. 202

Telegrams : Instrumen
Budapest

Metrimpeex



Electronic measuring instruments for all branches of science and industry



Radio-technical measuring
instruments



Telephone and telecommuni-
cation measuring instruments



Micro-wave measuring instruments



Measuring instruments for indust-
rial and scientific applications



Elements for automatic control



Measuring instruments for nuclear
physics



EXPORTERS:



METRIMPEX

HUNGARIAN TRADING COMPANY FOR INSTRUMENTS
LETTERS: BUDAPEST 62, P. O. B. 202.
TELEGRAMS: INSTRUMENT BUDAPEST

Wir empfehlen
aus unserer

Verlagsproduktion:

M. GÁSPÁR

Bibliographie der ungarischen chemischen Literatur 1926—1945

Budapest, 1957, 320 S. Format 17x24 cm. Geb.

A. GELEJI

**Die Berechnung der Kräfte und des Arbeitsbedarfs
bei der Formgebung im bildsamen Zustande der Metalle**

Zweite, verbesserte und erweiterte Auflage.
Budapest, 1955, 416 S., mit 400 Abb.
Format 17x24 cm. Geb.

E. MOSONYI

Wasserkraftwerke. Bd. I.

Budapest, 1956, 872 S., 508 Abb. und 6 Beilagen.
Format 17x24 cm. Geb.
Bd. II. In Vorbereitung. Erscheint in 1958

**Deutsch-Ungarisches und Ungarisch-Deutsches
Wörterbuch der Technik und Wissenschaften**

Bd. I. Deutsch-Ungarischer Teil
Budapest, 1951, XVI, 1170 S. Format 17x24 cm. Geb.
Bd. II. Ungarisch-Deutscher Teil
Budapest, 1953, XVI, 944 S. Format 17x24 cm. Geb.

**Englisch-Ungarisches und Ungarisch-Englisches
Technisches Wörterbuch**

Bd. I. Englisch-Ungarischer Teil
Budapest, 1951, XI, 976 S. Format 20x29 cm. Geb.
Bd. II. Ungarisch-Englischer Teil
Budapest, 1957, 752 S. Format 20x29 cm. Geb.



VERLAG DER UNGARISCHEN AKADEMIE DER WISSENSCHAFTEN
BUDAPEST

Bestellbar in jeder guten Buchhandlung, oder bei

KULTURA, BUDAPEST 62, POB. 149

A Budapesti Műszaki Egyetem Periodica Polytechnica címen idegen nyelvű tudományos folyóiratot indított. A folyóirat három sorozatban — vegyészeti, villamossági, valamint gépész- és általános mérnöki sorozatban — jelenik meg, évente négyszer, sorozatonként egy-egy kötetben. Az egyes kötetek terjedelme 14—18 ív.

A Periodica Polytechnicában megjelenő tanulmányok szerzői az Egyetem tanári karából és tudományos dolgozóiból kerülnek ki. Főszerkesztő Dr. Csűrös Zoltán egyetemi tanár, akadémikus.

A folyóirat előfizetési ára sorozatonként és kötetenként 50,— Ft. Megrendelhető az Akadémiai Kiadónál (Budapest 62, Postafiók 440. NB. egyszámlaszám: 05-915-111-44), a külföld számára pedig a Kultúra Könyv és Hírlap Külkereskedelmi Vállalatnál (Bp. 62, Postafiók 149. NB. egyszámlaszám: 43-790-057-181), illetve a vállalat külföldi képviselőinél és bizományosainál.

I N D E X

<i>Bárány, N.</i> : Wide-Angle Image Forming Systems	105
<i>Frigyes, A.</i> : The Influence of the Layout and Dynamic Characteristics of Servomechanisms on the Temperature Conditions of Separately Excited D. C. Servomotors Used in the Servomechanisms	131
<i>Csáki, F.</i> : Short-Circuit Currents in Circuits Containing Series Capacitors	155
<i>Eisler, J.</i> : Electric Breakdown as a Probability Process	173
<i>Klár, J.</i> : Einige Gesichtspunkte zur Beurteilung von vergleichenden Wirtschaftlichkeitsberechnungen bei Ausnutzung verschiedener Energieträger	189
Book Review — Buchbesprechung	199
Industrial Review — Aus der Industrie	203